Planning of inspections of fatigue-prone airframe

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Abstract

Switching to double inspection frequency after first fatigue crack discovery during aircraft inspection in operation is investigated. The method of calculation of the probability of fatigue failure based on the use of Markov chains theory is offered. For inspection program development on a base of lifetime approval test result processing the use of p-set function and minimax approach is offered. In this case there is no necessity to look for dubious compromise choice of required reliability and confidence probability.
Keywords: p-set function, minimax approach, inspection program, and approval test.

1. Introduction

Inspection program development should be made on the base of processing of lifetime test result. Usually a confidence interval is used for lifetime distribution parameter estimation and then for the reliability estimation. It is always very difficult to find compromise choice of required reliability and confidence probability. But if we should process some approval test data, when we should make some redesign of the tested system if some requirements are not met, then, as it will be shown later, it is possible to use minimax approach which provides required reliability independently of unknown parameters of lifetime distribution without use a confidence probability. For this purpose the p-set function definition is used. Here we consider some example of p-set function application to the problem of development and control of inspection program. We make assumption that some Structural Significant Item (SSI), the failure of which is failure of a system under consideration, is characterized by a random vector (r.v.) (T_d, T_c), where T_c is critical lifetime (up to failure), T_d is service time, when some damage (fatigue crack) can be detected. So we have some time interval, such that if in this interval some inspection will be fulfilled, then we can eliminate the failure of the SSI. We suppose also that a required operational life of the system is limited by so-called Specified Life (SL), t_{SL}, when system is discarded from service.

2. P-set function definition

P-set function for random vector is a special statistical decision function, which, in fact, is generalization of p-bound for random variable, definition of which was introduced much earlier [1-5]. P- set function for random vector is defined in following way.

Let *Z* and *X* are random vectors of m and n dimensions and we suppose that it is known the class $\{P_\theta, \theta \in \Omega\}$ to which the probability distribution of the random vector $W=(Z, X)$ is assumed to belong . Of the parameter θ, which labels the distribution, it is assumed known only that it lies in a certain set Ω, the parameter space. If $S_Z(x) = \bigcup_i S_{Z,i}(x)$ is such set of disjoint sets of z values as function of *x* that

$$\sup_\theta \sum_i P(Z \in S_{Z,i}(X)) \le p$$

then statistical decision function $S_z(x)$ is p-set function for r.v. $\boldsymbol{Z}$ on the base of a sample $x=(x_1,...,x_n)$.

Later on the value *x*, observation of the vector *X*, would be interpreted as result of some test or (some times it is more convenient) as estimate $\hat\theta = \hat\theta(x)$ of parameter θ; *Z* would be interpreted as some random vector-characteristic of some SSI in service: for example, $Z = (T_d, T_c)$. For inspection program development the p-set function defines the sequence of inspection moments, which defines some set $S_z(x)$ of values of *r.v.* $Z = (T_d, T_c)$.

3. Inspection program development

By processing results of some special approval test (full-scale fatigue test of airframe, for instance), we can get estimate $\hat\theta$ of parameter θ. The problem is to find (in general case) a vector function $t(\hat\theta)$, where $t = (t_1, t_2, ..., t_n)$, t_i is time moment of ith inspection, $i=1,2,\ldots,n$, *n* is inspection number, $t_{n+1} = t_{SL}$, in such a way, that failure probability of SSI under consideration

$$p_f(\theta, t) = \Sigma^{n+1}{}_{i=1} P(T_{i-1} \le T_d < T_c < T_i),$$

does not exceed some small value ε:

$$\sup_\theta p_f(\theta, t) \le \varepsilon,$$

where $T_1,...,T_n$ are moments of inspections: r.v. $T = (T_1,...,T_n) = t(\hat\theta)$; $T_0 = 0$; $T_{n+1} = t_{SL}$. This means that vector function $t(\hat\theta)$ in fact defines some p-set function for vector (T_d, T_c) at $p=\varepsilon$.

Usually we put $t_i = t_1 + d(i-1)$, $d = (t_{SL} - t_1)/n$, $i = 1,2,..., n+1$. Then we should choose only t_1 and n. For simplicity purpose we put $t_1 = d$ (in general case t_1 can be chosen, for example, as parameter-free p-bound for T_c, or we can try to get minimum of expectation value of n at fixed required reliability, etc) . Now probability of failure will be function of θ and n and we'll denote it by $p_f(\theta,n)$. We suppose existence of some $n_0 = n_0(\theta)$ such that for $n > n_0$ the function $p_f(\theta,n)$ monotonically decreases when n increases and $\lim_{n\to\infty} p_f(\theta,n) = 0$ for all θ . Let $n(\theta,\varepsilon)$ is minimal inspection number n such that $p_f(\theta,n) \le \varepsilon$, where ε is some small value. But true value of θ is not known. So $\hat{n} = n(\hat{\theta},\varepsilon)$ and $\hat{p}_f = p_f(\theta,\hat{n})$ are random variables. We suppose, that we begin the commercial production and operation only if some specific requirement to reliability are met. Let us denote in general case this event as $\hat{\theta} \in \Theta_0$, where $\Theta_0 , \Theta_0 \subset \Omega$, is some part of parameter space. We suppose, that if $\hat{\theta} \notin \Theta_0$ (in this paper we suppose that $\hat{\theta} \notin \Theta_0$ if required inspection number for some fixed ε exceeds some threshold $n_{\max}$ or estimate of expectation value of T_c is too small in comparison with t_{SL}), then we make redesign of the SSI in such a way, that probability of failure after this redesign will be equal to zero.

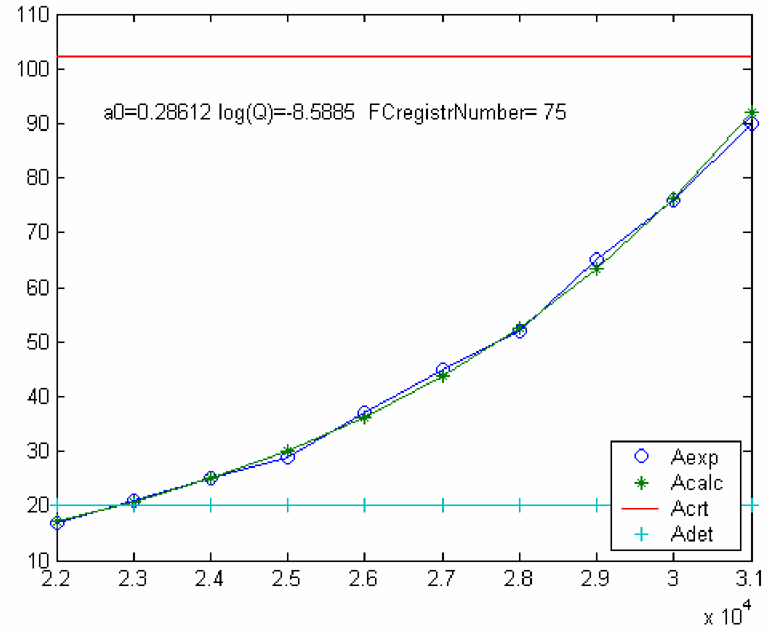

Fig.1. **Exponential model for experimental data approximation**

Let us define

$$\hat{p}_{f0} = \begin{cases} p_f(\theta,\hat{n})\, if \;\; \hat{\theta} \in \Theta_0, \\ 0 \;\; if \;\; \hat{\theta} \notin \Theta_0. \end{cases}$$

For this type of strategy the mean probability of fatigue failure $w(\theta,\varepsilon) = E_\theta(\hat{p}_{f0})$ is a function of θ and ε . If for limited t_{SL} it has a maximum, depending on ε then the choice of maximal value of $\varepsilon = \varepsilon^*$ for which $\max_\theta w(\theta,\varepsilon^*) \le 1 - R$ and inspection number $n = n(\hat{\theta},\varepsilon^*)$ is such strategy for which required reliability R is provided.

4. Numerical example

The simplest example of considered approach with unchangeable interval between inspections is given in [5]. The disadvantage of this strategy is a large number of inspections in the initial period when the probability to discover the fatigue crack is negligibly small. In this paper we consider more complex strategy when the inspection program, preliminary developed on the base on approval test information, later on can be changed after discovery of first fatigue crack.

The numerical calculations will be based on exponential approximation of fatigue crack growth function when the size, $a(t)$, of fatigue crack is described by equation $a(t) = a(0)\exp(Qt)$. Example of this type approximation of experimental data is shown in Fig.1. Then

$$T_d = (\log a_d - \log a_0)/Q = C_d / Q ,$$
$$T_c = (\log a_c - \log a_0)/Q = C_c / Q ,$$

where a_0 is $a(0)$, a_d is a crack size, when the probability to discover it is equal to unit, a_c is a crack size, which corresponds to the maximum residual strength of an aircraft component allowed by special design regulation, T_d is a time for crack to growth by its detectable size and T_c is a time for crack to growth by its critical size. In the simplest case let us suppose that a_0, a_d and a_c are constants. Usually it is assumed that random variable $\log(T_C) = \log(C_C) - \log(Q)$ has normal distribution. This means that $\log(Q)$ has normal distribution also. Suppose in

	E_1	**E_2**	**E_3**	**…**	**E_{n-1}**	**E_n**	**E_{n+1}**	**E_{n+2} (SL)**	**E_{n+3} (FF)**	**E_{n+4} (CD)**
E_1	0	u_1	0	…	0	0	0	0	q_1	v_1
E_2	0	0	u_2	…	0	0	0	0	q_2	v_2
E_3	0	0	0	…	0	0	0	0	q_3	v_3
…	…	…	…	…	…	…	…	…	…	…
E_{n-1}	0	0	0	…	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	…	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	…	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	…	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	…	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	…	0	0	0	0	0	1

Fig.2. **Matrix of transition probabilities**

operation there is a park of N aircraft of the same type. And we choose the following strategy. We developed two inspection programs with n and $(2n+1)$ inspections. We begin the operation of the park, using first program. But after at least one crack discovery we two times decrease interval between inspections on remains of aircraft park and continue the operation of every aircraft up to specified life but this time independently one from another. We suppose that after retrofit of aircraft on which the fatigue crack was discovered the probability of its failure up to specified life will be equal to zero. Let us refer to this strategy as SWn-strategy as distinct to WSn-strategy without frequency of inspections change.For failure probability calculation we need to use some results of Markov Chains theory. We define the set of states in following way. Let us denote the service of aircraft in certain ith interval (t_{i-1},t_i) as a state E_i. For all $i \leq (n+1)$ there are three possible transitions from this state to another states, which are represent (1) transition into next (i+1)th time interval or, if i=n+1, successful end of service(absorbing state E_{n+2} (SL-state)), (2) transition into absorbing E_{n+3} state (FF-state), corresponding to the fatigue failure, and (3) transition into absorbing E_{n+4} state (CD-state), corresponding to discovery of the fatigue crack. The corresponding probabilities we'll notate by u_i, q_i, v_i correspondingly. For all three absorbing states there are units in main diagonal. All the others probabilities of the considered matrix of transition probabilities are equal to 0. The corresponding matrix of transition probabilities is shown in Fig.2. If random variable $\ln(Q)$ has normal distribution $N(\theta_0,\theta_1^2)$ then conditional probabilities u , q_i are defined by formulas

$$u_i = a_i / a_{i-1},$$

$$q_i = \max(0,(a_{i-1} - b_i)/(1 - a_{i-1})),$$

where

$$a_i = \Phi\left(\ln(C_d / t_i) - \theta_0\right)/\theta_1),$$

$$b_i = \Phi\left(\ln(C_c / t_i) - \theta_0\right)/\theta_1),$$

$\Phi(.)$ is distribution function of standard normal variable. It is clear that $v_i = 1 - u_i - q_i$.

It is necessary to mention, that if we consider a park of N aircraft of the same type and if we are interested to know the probabilities of the failure of at least one aircraft or crack discovery in at least one aircraft of the park then instead of q_i and u_i we should use $q_{i,N} = 1-(1-q_i)^N$ and

$u_{i,N} = (u_i)^N$. Let us denote this type of matrix by P(n,N,WS). In order to study SWn-strategy we need also matrix of P(n,N,SW) type. It is a matrix with *n* additional "absorbing" states, corresponding (after first fatigue crack discovery at time moment t_i, $i = 1,2,...,n$) to transition into the "secondary" process defined by the matrix P((2n+1),1,WS) with (2*i*+1)th initial state.

Now, for the simple example, let us consider SWn-strategy with two initial number of inspections: *n*=2. The corresponding possible transitions in this case are shown in Fig. 3: after switching to doubled frequency the remaining time intervals are splits into two parts. In this case in the matrix P(2,N,SW) there are two additional (in comparison with P(2,N,WS) matrix) absorbing states *CD*1 and *CD*2 (see Fig.4), corresponding to states EE_3 and EE_5 (see Fig. 3) from the matrix P(5,1,WS). The states CD1, CD2 are absorbing states corresponding to "absorption" at the inspection 1 and inspection 2.

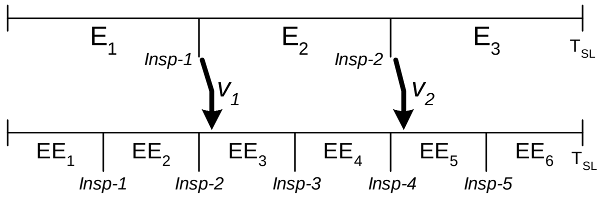

Fig. 3. **Switching to the double inspection frequency state graph, 2-inspections initial model**

The structure of considered matrices can be described in following way:

Q	*R*
0	*I*

where *I* is matrix of identity corresponding to absorbing states, 0 is matrix of zeros. Then matrix of probabilities of absorbing in different absorbing states for different initial transient states is defined by formula

$$B = (I - Q)^{-1} \cdot R$$

	E₁	**E₂**	**E₃**	**E₄ (SL)**	**E₅ (FF)**	**E₆ (CD)**	**E₇ (CD 1)**	**E₈ (CD 2)**
E₁	0	$u_{1,N}$	0	0	$q_{1,N}$	0	$v_{1,N}$	0
E₂	0	0	$u_{2,N}$	0	$q_{2,N}$	0	0	$v_{2,N}$
E₃	0	0	0	$u_{3,N}$	$q_{3,N}$	$v_{3,N}$	0	0
E₄ (SL)	0	0	0	1	0	0	0	0
E₅ (FF)	0	0	0	0	1	0	0	0
E₆ (CD)	0	0	0	0	0	1	0	0
E₇ (CD1)	0	0	0	0	0	0	1	0
E₈ (CD2)	0	0	0	0	0	0	0	1

Fig. 4. **The matrix P(2,N,SW) for SW2-strategy**

In general case by the use of relevant formulas for absorbing Markov Chains we can calculate the probabilities of absorbing in relevant states of the matrices *P*(*n*,*N*,*WS*), *P*((2*n*+1),1,*WS*) and then using *P*(*n*,*N*,*SW*) matrix we can calculate total probability of failure (absorbing probability in state FF) for the *SWn*-strategy .

Let $b(i, j \mid P)$ be item in i^th^ row in j^th^ column of matrix B corresponding to the matrix P. For example $b(1, j \mid P(n, N, SW))$ is j^th^ item of first row of matrix B, corresponding to matrix $P(n, N, SW)$, and it denotes the probability of absorption in state $E_{(n+1+j)}$,

$j = 1,2,...,n+3$, in the process defined by matrix $P(n,N,SW)$. Specifically, for $j = 1,2,3$ it is probability of absorption in states SL, FF, CD correspondingly. Probabilities $b(1,4 \mid P(n,N,SW))$, $b(1,5 \mid P(n,N,SW))$, are probabilities of "absorption" in states D1 , D2, of the process, defined by matrix $P(n,N,SW)$. Probability $b(2i+1,2n+3 \mid P(n,1,WS))$ denotes the probability of absorption in state FF if the inspection frequency change takes place after i inspections in a process defined by matrix $P(n,N,SW)$.

The average number of failure in the park will be equal probability of failure (multiplied by unit) before "switching" to doubled frequency of inspections and probability of failure of one aircraft after this moment multiplied by (N-1):

$$E(N_f) = b(1,2 \mid P(n,N,SW)) + \sum_{i=1}^{n} b(1,3+i \mid P(n,N,SW)) + (N-1)b(2i+1,2 \mid P(2n+1,1,WS))).$$

Probability of failure of one aircraft $p_f = E(N_f)/N$. By the use of Monte Carlo method for modeling $\hat{\theta}_0$ we can calculate the function $w(\theta,\varepsilon) = E_\theta(\hat{p}_{f0})$, the average probability of failure of one aircraft in the park of N aircraft for *SWn* –strategy. Example of the calculation of the function $w(\theta,\varepsilon)$, the probability of redesign, the reliability without inspection as function of $(\theta_0 - \hat{\theta}_0)/\theta_1$ and corresponding initial data are shown in Fig.5. Let us remind that $\hat{\theta}_0$ can be considered as estimate of speed of fatigue crack growth (in log-scale) and in considered example we have event $\hat{\theta} \notin \Theta_0$, if

1) for ε_1=0.0001 (in this case approximate value of probability of at least one failure in park $\varepsilon = \varepsilon_1 N$=0.01) the required number of inspections $\hat{n} = n(\hat{\theta},\varepsilon)$ is more than 3

or

2) estimate of mean T_c lesser than t_{SL} =40000 (flights).

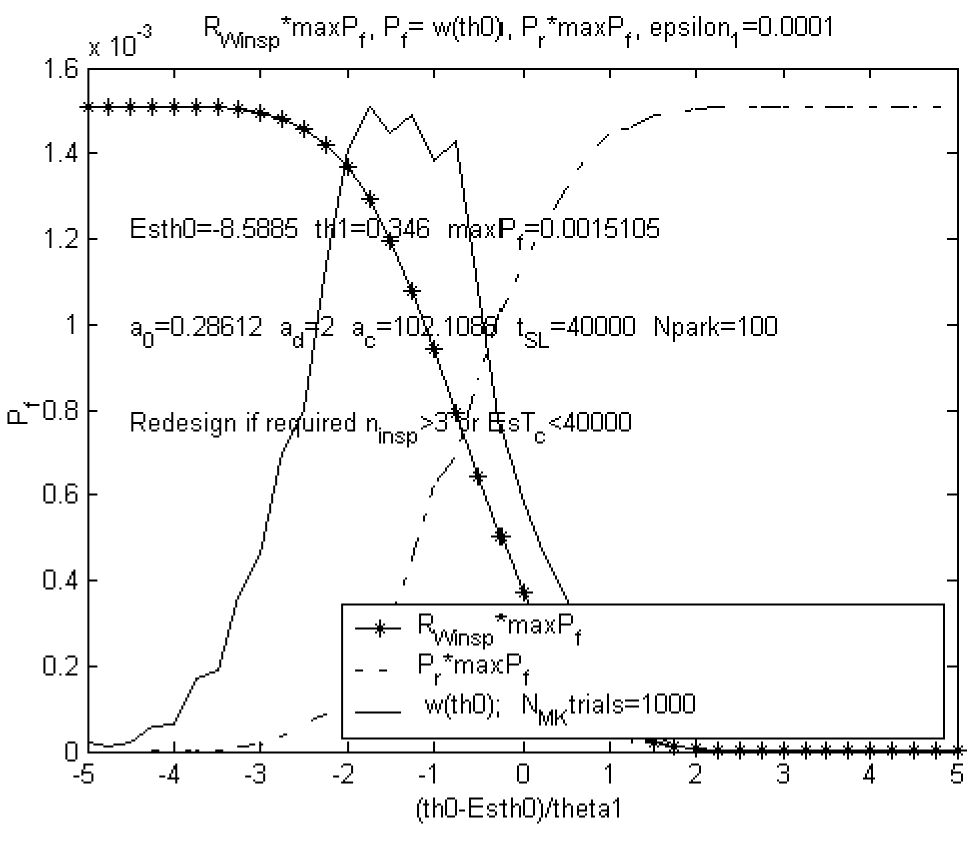

Fig. 5. **Function $w(\theta,\varepsilon) = E_\theta(\hat{p}_{f0})$ for ε_1 =0.0001, EsTc is estimate of $E(T_c)$ (Because of MATLAB-plot limitation instead of θ_0 in the figure the "th0", instead of $\hat{\theta}_0$ the abbreviation Esth0, instead of θ_1 the th1 and theta1 are used !). N_{MK} trials is the number of Monte Carlo trials (MC - sample size of $\hat{\theta}_0$ for every θ_0), P_r is redesign probability, R_{Winsp} is reliability without inspections**.

The maximum of the function $w(\theta,\varepsilon)=E_\theta(\hat{p}_{f0})$ for $\varepsilon=\varepsilon_1 N$=0.01 is equal to 0.0014. (It is worth to mention, that the maximum of this function exists because we make redesign of "week" structural significant item (when "speed" of fatigue crack growth, θ_0 is too high) and, on the other side, we do not need any inspection if structural significant item is too strong (when θ_0 is too small). So if required reliability (of one aircraft) is equal to 0.9986 then we for the *SWn*-strategy we should choose the inspection number for $\varepsilon=N\varepsilon_1$ =0.01. For the considered example $\hat{n}=n(\hat{\theta},\varepsilon\ )$ =2.

In this paper we have considered the strategy of two time decreasing of inspection interval at every discovery of fatigue crack in first period of operation. But similar calculation can be made if at every CD-event some specific strategy will be used. But this is subject of another paper.

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Received 04 04 2006