ISSN 1392-2114 ULTRAGARSAS, Nr.2(55). 2005.

Simulation of the pitch-catch of ultrasound waves at oblique incidence to the plane glass layer

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Introduction

When developing and spreading modern materials and technologies the parameters of layer structures are sufficiently widely investigated. Especially for that purpose ultrasonic NDT methods are successfully used [1]. Though the ultrasonic measuring methods of parameters of layers are well developed and widely used, but applying them in practice some difficulties appear. Some of them are related to the possibility to get measuring information only from one side of the layer structure [2,3]. At that case, when performing the measurements of parameters of multi-layer structures by ultrasound, not always it is possible to obtain information about the parameters of all layers. This is stipulated by the losses of ultrasound signals in the separate layers as well as by the losses at the boundaries between them. These losses depend on the differences of acoustical impedances. Therefore investigation of such multi-layer structures like as plate type heat exchanges, highly insulating glazing units of windows and multi-layer composite materials is limited by measurement of parameters of separate few layers [2-4].

Other difficulties appear because the acoustical properties of the material often are not exactly known. For that reason the measurement of thickness and other parameters of the layer is problematic. The problems are related to the fact that the velocities of propagation of acoustic waves of different types in the medium under investigation usually are unknown. In such a case the determination of thickness of the layer is possible only by using of two separate measuring channels. At least in one channel the layer must be irradiated at an angle to its surface. However, when sounding at the angle to the surface of the layer, the longitudinal, shear and other types of ultrasound waves are excited [5]. The velocities of propagation of different types of ultrasound waves usually are unknown exactly too. So, when measuring the thickness of a layer by ultrasound, the reflections of signals from the different surfaces of the layer are used. In that case the acoustic impedances of transition layer (prism) of a transducer and the media under investigation are not matched. Since the velocity of propagation of ultrasound waves is not known exactly, the angles of propagation and reflection of ultrasound waves of different types are not known exactly too. It is not clear at what angles it is reasonable to sound the layer and what types of ultrasound waves to use for measurements. Therefore, the aim of this investigation was to determine the dependence of coefficients of transmission and reflection of ultrasound waves on the angle of incidence to the layer structure. The possibilities of receiving of measuring signals, when the measurements are performed from one side of the layer, are being investigated too by modeling. The range of change of the angles of incidence was chosen sufficiently wide. It is related to the possibility to evaluate the change of the magnitude of the energy of longitudinal or shear waves transmitted through the interface of two materials with different impedances, when the angle of incidence is changed. The results obtained will be useful for application of longitudinal or shear waves for measurement of the parameters of layers. The dependence of the reflections coefficient on the angle of incidence enables one to evaluate the distribution of reflected waves in the prism of a transducer. These waves stipulate the reverberation noise of piezoelectric transducers and are very important characteristic of transducers, especially used for investigation of multi-layer structures.

Theoretical investigation

Let the plane ultrasound wave (longitudinal-I_d or shear-I_s) is propagating in an isotropic elastic material and reaches the plane interface with the other material, which distinguishes by acoustic properties. The incidence wave, due to a mode conversion at the boundary between the both materials, is transformed to 2-4 independent waves (Fig.1):

reflected longitudinal – R_d;
transmitted longitudinal – T_d;
reflected shear – R_s;
transmitted shear – T_s.

Let us assume that the boundary contact between the both materials is rigid. Then the boundary conditions will be [6]:

1) corresponding components of the displacement of

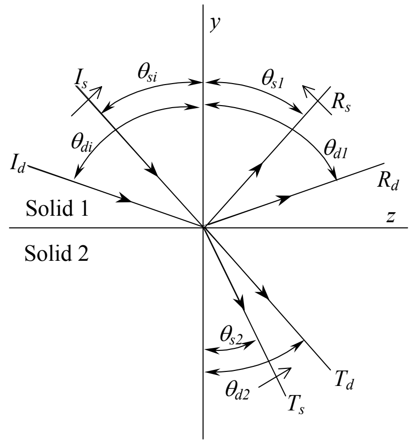


Fig.1. **Ultrasound wave interaction with a solid-solid interface**

particles are equal among themselves:

$$u_y^{(2)} = u_y^{(1)}\,;\; u_z^{(2)} = u_z^{(1)}\,.$$

2) the components of the one-named stress tensors of both materials are equal among themselves too:

$$\tau_{yy}^{(2)} = \tau_{yy}^{(1)}\,;\; \tau_{zy}^{(2)} = \tau_{zy}^{(1)}\,.$$

The indexes above designates the material (1-from which the wave incidence, 2–to which the wave incidence).

Then, in accordance with [6], it may be written

$$\begin{bmatrix} -u_y^{(d1)} & u_y^{(d2)} & -u_y^{(s1)} & u_y^{(s1)} \\ -u_z^{(d1)} & u_z^{(d2)} & -u_z^{(s1)} & u_z^{(s2)} \\ -z_{yy}^{(d1)} & z_{yy}^{(d2)} & -z_{yy}^{(s1)} & z_{yy}^{(s2)} \\ -z_{zy}^{(d1)} & z_{zy}^{(d2)} & -z_{zy}^{(s1)} & -z_{zy}^{(s2)} \end{bmatrix} = \begin{bmatrix} u_y^{(i)} \\ u_z^{(i)} \\ \tau_{yy}^{(i)} \\ \tau_{zy}^{(i)} \end{bmatrix}. \qquad (1)$$

The index i show that the parameters are related to the incidence wave.

Eq. 1 can be written by using the displacement amplitudes as follows

$$[a_{kl}]\cdot\begin{bmatrix} R_d \\ T_d \\ R_s \\ T_s \end{bmatrix} = [b_k], \qquad (2)$$

depending on whether longitudinal or shear wave incidence is considered.

The values of the coefficients of the matrix $[a_{kl}]$ and $[b_k]$ are determined by using formulas:

$$\tau_{yy} = \lambda\frac{\partial u_z}{\partial z} + (\lambda + 2\mu)\frac{\partial u_y}{\partial y}\,,$$

$$\tau_{zy} = \mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right),$$

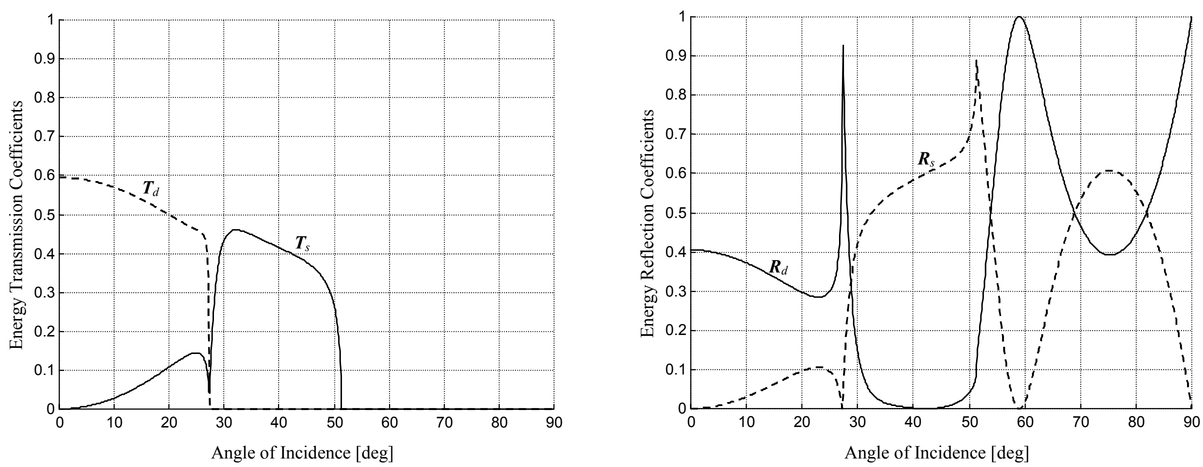
$$\mu = \rho c_s^2\,;\; \lambda + 2\mu = \rho c_d^2\,.$$

When knowing the values of coefficients, included in Eq.2, the reflection (R_d and R_s) and transmission (T_d and T_s) of coefficients of ultrasound wave may be determined. After the determination of displacement coefficients of the wave transmission and reflection it is easy to determine the coefficients of the wave energy reflection (***R****d* and ***R****s*) and transmission (***T****d* and ***T****s*). By analogy, the coefficients of reflections may be determined for the solid gas media interface.

When modeling, the first material, from which the ultrasound wave was radiated, was the plexiglass. Its density ρ=1170 kg/m^3, the velocity of longitudinal waves c_l=2650 m/s and the velocity of shear waves c_s=1335 m/s [7]. The second layer (layer under investigation) was the glass, which density ρ=2440 kg/m^3, the velocity of longitudinal waves c_l=5770 m/s and the velocity of shear waves c_s=3392 m/s [5]. The second boundary of the glass layer is the interface with gas (air), from which the ultrasound waves are reflected. Its density ρ=1.204 kg/m^3, the velocity of longitudinal waves c_l=343.37 m/s (20^o C) [8]. Because calculation of the coefficients of ultrasound energy transmissions and reflections through all these structures is complicated, the simulation at once was performed for separate interfaces by order. The dependencies of coefficients of energy transmission and reflection on the angle of incidence of longitudinal $\boldsymbol{T_d}$ and shear $\boldsymbol{T_s}$ waves were calculated by modeling.

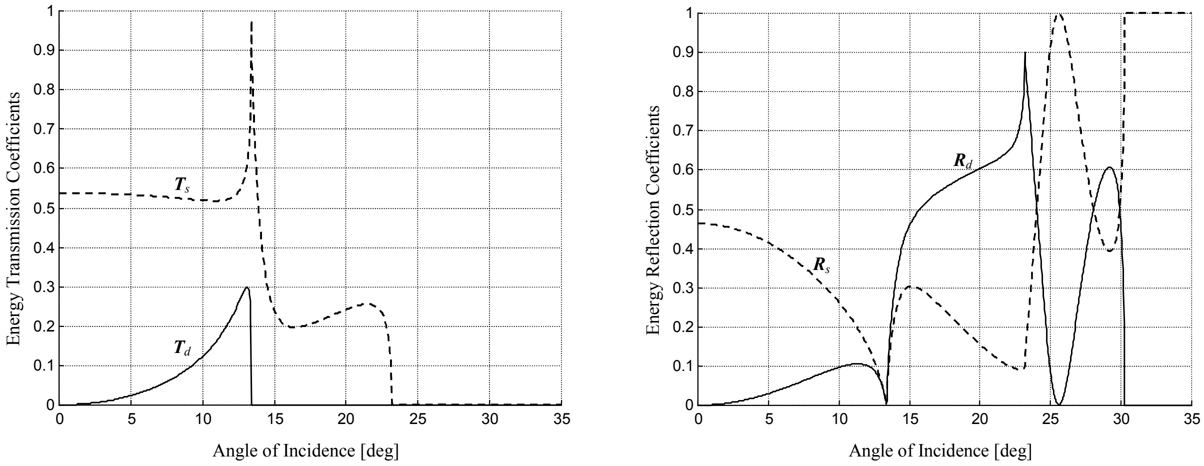
Results and discussion

The variation of transmission and reflection coefficients of ultrasound energy through the plexiglass-glass interface, when the angle of incidence was increased and the transducer was excited by longitudinal waves, is shown in Fig.2a,b. How it is seen from Fig.2a, the energy of the longitudinal wave, transmitted from the plexiglass to glass, was slightly decreasing when the angle of incidence of longitudinal waves was increased till the first critical angle. However, the part of energy, transformed to the shear waves increases respectively. When the angle of incidence of longitudinal waves was between the first and second critical angle, the longitudinal waves were not transmitted to the glass (Fig.2a). All energy of the longitudinal waves was transformed to the shear waves ($\boldsymbol{T_s}$ in Fig.2a and $\boldsymbol{R_s}$ in Fig. 2b). It is to be noted that in the



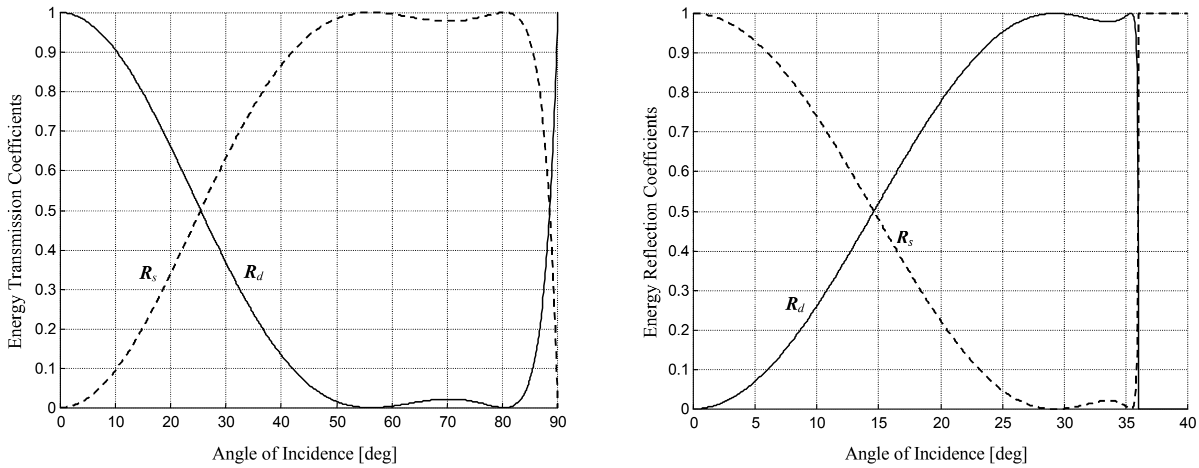
a b

Fig.2. **The dependencies of energy transmission coefficients T_d, T_s (a) and energy reflection coefficients R_d, R_s (b) on the angle of incidence for plexiglass-glass interface and longitudinal excitation**



a b

Fig.3. **The dependencies of energy transmission coefficients T_s, T_d (a) and energy reflection coefficients R_s, R_d (b) on the angle of incidence for plexiglass-glass interface and shear excitation**



a b

Fig.4. **The dependencies of energy reflection coefficients at a glass-gas media interface on the angle of incidence for longitudinal excitation (a) and shear excitation (b)**

vicinity of the first critical angle, the coefficient of the energy transmission of ultrasound waves is approaching to zero, but the reflection coefficient is approaching to unity. Longitudinal and shear waves reflected from the plexiglass-glass interface in reality are reflected to the prism of electroacoustical transducer. Therefore, when developing wedge type transducers, it is necessary to evaluate the types of reflected waves. This enables to reduce the reverberation noise of transducers and to increase the accuracy of measurements.

In analogy the calculations were performed for the case of the shear waves, polarized in the plane of the angle of incidence. The transmission coefficient of the shear wave from the plexiglass to the glass remains almost constant (Fig.3a), when the angle of incidence in plexiglass is increased till the first critical angle. Besides that, two longitudinal waves are excited. The transmission coefficient of the first longitudinal wave $\boldsymbol{T_d}$ and reflection coefficient $\boldsymbol{R_d}$ of the second longitudinal wave are increased with the angle of incidence (Fig.3a and 3b, respectively). But when the angle of incidence is between the first and the second critical angles, especially significant is the growth of the longitudinal wave ($\boldsymbol{R_d}$ in Fig.3b), reflected to the plexiglass.

When longitudinal or shear waves are excited in the plexiglass and the angle of incidence is changed, significant part of the ultrasound energy is transferred to the glass and achieves the second interface glass-gas medium. Because the acoustical impedance of gas medium is very small, almost all ultrasound energy is reflected from the glass-gas media interface (Fig.4a,b). In addition, the reflected waves undergo the mode conversion, when the angle of incidence is increased, and returns to the boundary glass-plexiglass (Fig. 5ab). At this boundary the transmission coefficient ($\boldsymbol{T_d}$) of longitudinal wave is slowly decreasing, but the transmission coefficient of the shear waves ($\boldsymbol{T_s}$ in Fig. 5a) is increasing.

When the angle of incidence of the shear wave in the glass is less than the first critical angle more than a half energy of the shear waves is transferred from the glass

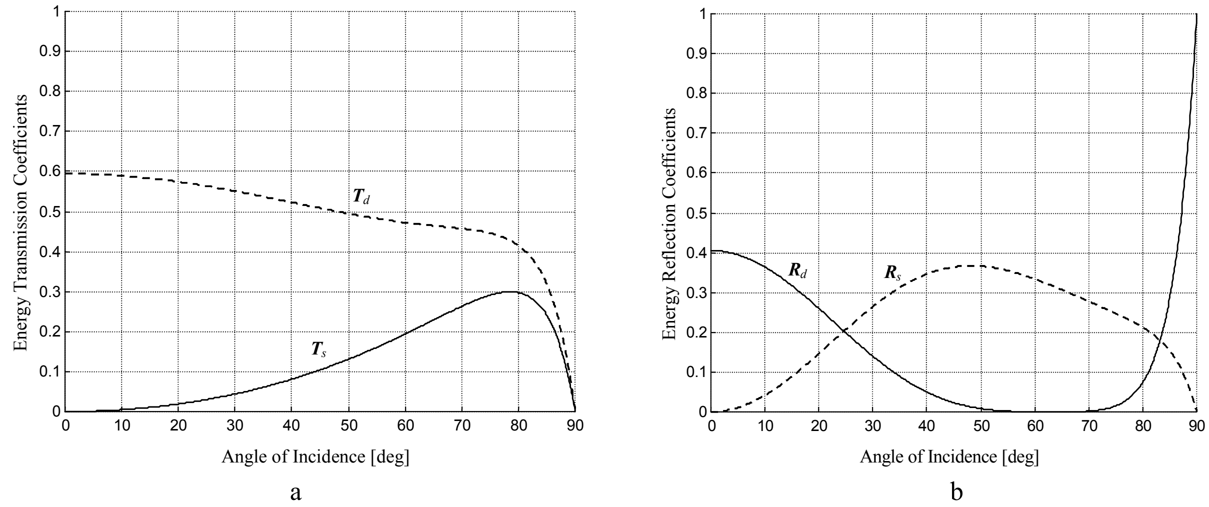


Fig.5. **The dependencies of energy transmission coefficients T_d, T_s (a) and energy reflection coefficients R_d, R_s (b) on the angle of incidence for glass-gas media interface and longitudinal excitation**

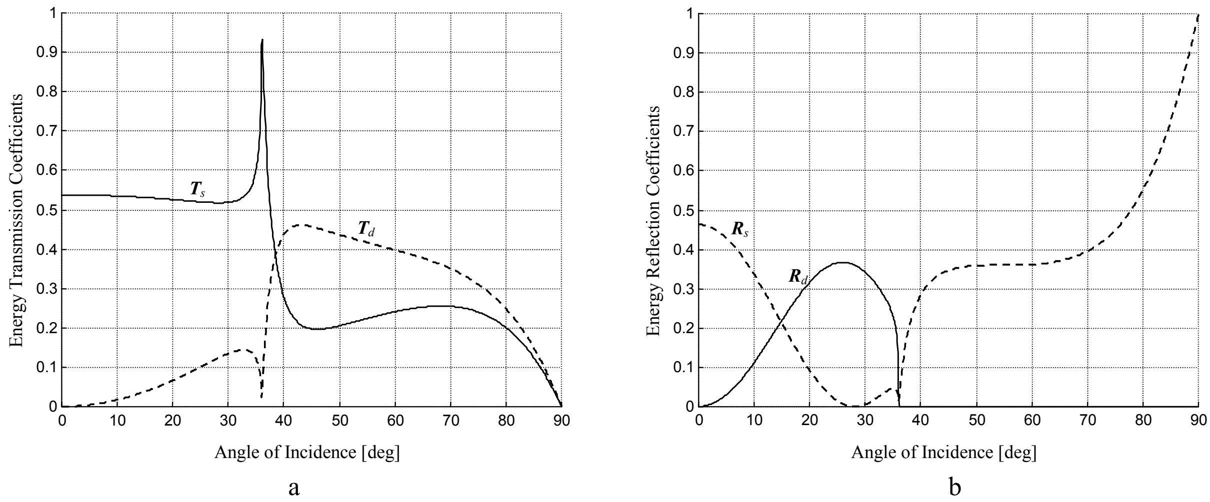


Fig.6. **The dependencies of energy transmission coefficients T_s, T_d (a) and energy reflection coefficients Rs, Rd (b) on the angle of incidence for glass-gas media interface and shear excitation**

layer to the plexiglass (Fig.6a,b). Besides that, the longitudinal wave (T_d in Fig.6a) is generated in the plexiglass due mode conversion. Its amplitude is increased if the angle of incidence of shear wave is between the first and second critical angle. At that case, the reflection coefficient R_s of the shear wave is significantly increased too (Fig.6b).

How it is seen from analysis presented here, the dependencies of transmission and reflection coefficients of ultrasound waves on the angle of incidence are very complicated. From the energy transmission coefficients through separate boundaries it is not easy to determine the energy transmission coefficient in a measuring channel, which consists of plexiglass-glass, glass-gas media-glass and glass-plexiglass. Therefore the energy transmission and reflection coefficients through all these boundaries (Fig.7) were calculated. The symbols of indexes of ultrasound waves energy transmission and reflection coefficients in turn show: 1-rst, the type of the incidence wave till the interaction with the first interface; 2-nd, the type of the wave after the first interaction with the first interface; 3-rd, the type of the wave after the reflection from glass-air interface; 4-th, the type of transmitted or reflected wave after the interaction with the glass-plexiglass interface. The index *d* shows that it is the longitudinal wave and *s* is the shear wave. The results of calculation are shown in Fig.8 and Fig.9. How is seen in

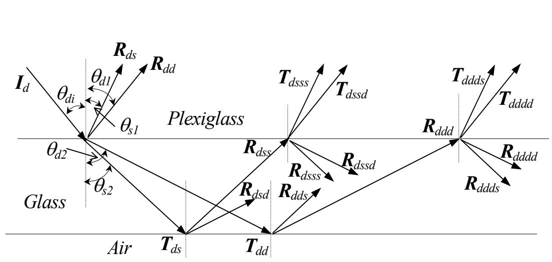


Fig.7. **Ultrasound wave interaction with a solid-solid, solid-gas-solid and solid-solid interfaces at oblique incidence**

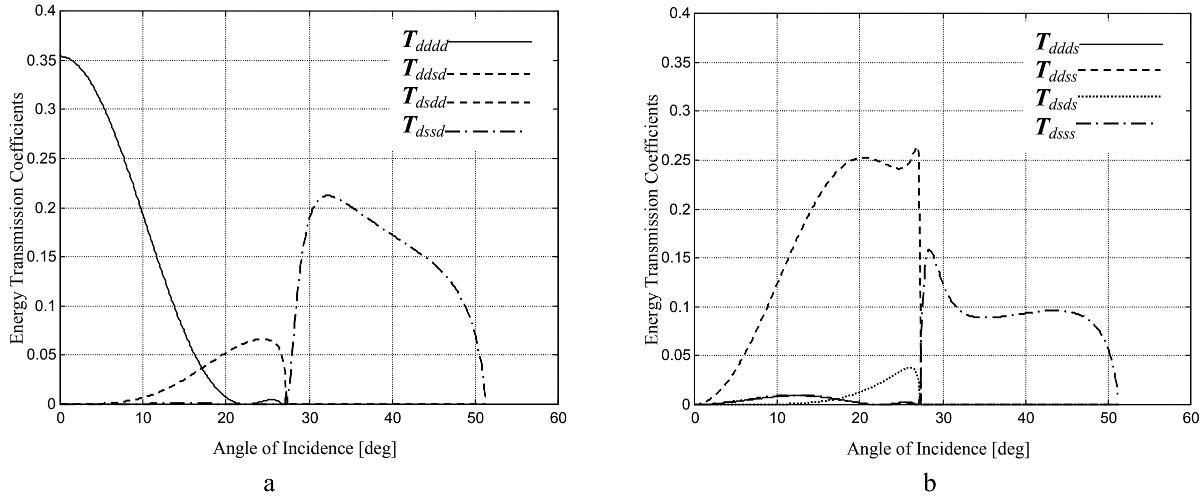


Fig.8. **The dependencies of energy transmission coefficients through the plexiglass-glass, glass-gas media-glass and glass-plexiglass structure on the angle of incidence for longitudinal excitation, when longitudinal waves are received (a), shear waves are received (b)**

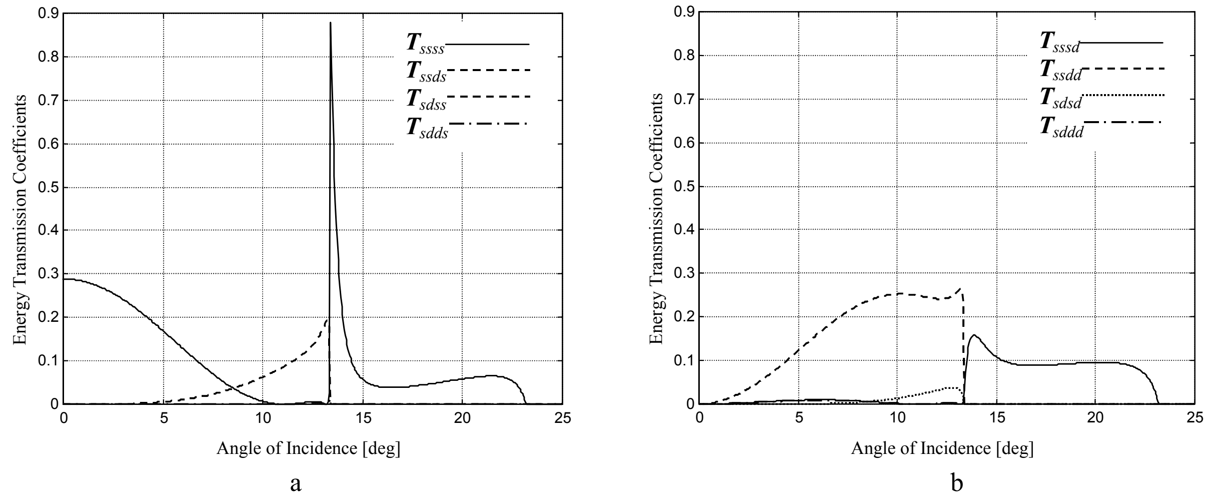


Fig.9. **The dependencies of energy transmission coefficients through the plexiglass-glass, glass-gas media-glass and glass-plexiglass structure on the angle of incidence for shear excitation, when shear waves are received (a) and longitudinal waves are received (b)**

Fig.8a, when the angle of incidence is changed in narrow range (till 10°) it is reasonable to use the longitudinal waves. But the energy transmission coefficient ($\boldsymbol{T}_{dddd}$) through all this structure is rapidly decreasing with the angle of incidence. If the angle of incidence exceeds 12° it is purposive to transmit to the glass layer the longitudinal waves, but to receive the shear waves ($\boldsymbol{T}_{ddss}$), generated at a boundary glass- gas media and reflected to the glass media (Fig.8b). These shear waves pass the glass-plexiglass interface and may be received till the angle of incidence of longitudinal waves in the plexiglass approaches to the first critical angle (θ=28° in Fig.8a). So, when the longitudinal waves are excited in the plexiglass and transmitted to the glass layer, the shear waves ($\boldsymbol{T}_{ddss}$) generated by mode conversion at the glass-gas media interface and reflected to the glass-plexiglass boundary may be received. When the angle of incidence of longitudinal wave is increased more than the first critical angle ($\theta \geq 30°$ for the plexiglass-glass interface) the longitudinal waves are not excited in the glass layer entirely. A significant part of the energy of longitudinal waves by the mode conversion at the boundary plexiglass-glass is converted to the shear waves. These waves are reflected from the glass-gas interface and returns as a shear waves to the glass-plexiglass boundary. At this boundary a new transformation of the shear waves to the longitudinal occurs ($\boldsymbol{T}_{dssd}$ in Fig.8a). Therefore the longitudinal waves are excited and received in the plexiglass, but the shear waves pass through the glass layer under investigation in both directions ($\boldsymbol{T}_{dssd}$).

The variations of the transmission coefficients of the shear wave energy through all these boundaries are shown in Fig. 9. When the angle of incidence is small (θ=5°…7°) it is reasonable to use the shear waves. But at that case the received energy is rapidly decreased ($\boldsymbol{T}_{ssss}$ in Fig.9a). Besides that, till the first critical angle, the energy transmission coefficients of the shear waves ($\boldsymbol{T}_{ssds}$ and $\boldsymbol{T}_{sdss}$), which undergo double conversion, are increased. In a vary narrow range over the first critical angle ($\Delta\theta \approx 1°$), the energy transmission coefficient of the shear waves is significantly increased. However, this theoretical possibility is to be confirmed by experimental investigations. The shear waves, at the interface glass-gas medium converted to the longitudinal waves, till the first critical angle may be received as longitudinal waves (T_{ssdd} in the Fig.9b). Over the first critical angle the shear waves, reflected from the glass-gas interface and converted at the boundary glass-plexiglass, may be received as longitudinal waves (T_{sssd}) too.

The results of investigation show that the best types of waves, which may be used for the measurement of plane glass layer parameters are: the T_{dssd}, T_{dsdd}, T_{ddds} and T_{dddd} waves for longitudinal excitation and T_{ssds}, T_{sdss}, T_{ssdd} and T_{sssd} for shear excitation.

Conclusions

The results of investigation show that longitudinal waves as well as shear waves may be used for determination of thickness and other parameters of layered structures, when the one-side ultrasound measuring methods are applied

How it is seen from the results of simulation, the dependencies of transmission and reflection coefficients of longitudinal and shear ultrasound waves on the angle of incidence are complicated enough. The energy transmission and reflection coefficients through the separate boundary enables one to evaluate the influence of reverberation noise, which occurs in the wedge of measuring transducers.

When the angle of incidence is small ($\theta < 10°$) the longitudinal waves as well as the shear waves may be applied for measurements. But the energy transmission coefficients of longitudinal (T_{dddd}) and shear (T_{ssss}) waves through all structure plexiglass-glass, glass-gas media-glass and glass-plexiglass are rapidly decreasing with the angle of incidence.

When the angle of incidence of longitudinal waves in the plexiglass is approaching to the first critical angle, the shear waves, generated by the mode conversion at the glass-gas medium interface and reflected to the glass-plexiglass boundary (T_{ddss}), may be received. Over the first critical angle the longitudinal waves are not excited in the glass layer entirely. Therefore, the longitudinal waves are excited and received in the plexiglass layer, but the shear waves (T_{dssd}) pass through the glass layer in both directions.

When the angle of incidence of the shear waves in the plexiglass is approaching to the first critical angle, the energy transmission coefficients of the shear waves, which undergo double conversion (T_{ssds} and T_{sdss}), are increased. The shear waves at the interface glass-gas medium converted to the longitudinal waves, till the first critical angle may be received as longitudinal waves (T_{ssdd}). Over the first critical angle the shear waves undergo the mode conversion at a glass-plexiglass interface and may be received as longitudinal waves (T_{sssd}) too.

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Kampu žadinamų ultragarso bangų peratspindžio modeliavimas plokščiajame stiklo sluoksnyje

Reziumė

Sumodeliuotos ultragarso bangų, perėjusių per organinio stiklo ir stiklo ribą, atsispindėjusių nuo stiklo ir oro ribos ir dar kartą perėjusių per stiklo ir organinio stiklo ribą, priėmimo galimybės. Išnagrinėti plokščiųjų (išilginių ir skersinių) ultragarso bangų transformacijos ir energijos pasiskirstymai šiose ribose, kai žadinimo kampai kinta nuo 0° iki 90°. Parodytos ultragarso bangų energijos perdavimo per tris minėtas ribas koeficientų priklausomybės nuo bangų transformacijos šiose ribose ir žadinančios bangos išspinduliavimo kampo.

Pateikta spaudai 2005 05 08