Modelling of 3D reflections from triangles using the Huygens approach

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Abstract:
Objective of this study was to develop an acoustic computer model, which would enable to calculate the signals reflected by triangles arbitrally oriented in space. Modelling of 3D reflections from triangles using the Huygens approach was performed. The rotation of the triangle in such a way, that after the turn the triangle would be located in one plane allowed to reduce the amount of data and, consequently, to increase the speed of calculations. Modelling was performed at different triangle angles with respect to a transducer. The performed simulations show, that the triangle as a shape can be recognized only in the case of a specular reflection.
Keywords: 3D modelling, ultrasonic, Huygens.

Introduction

Often one of the ways to solve complicated inspection tasks is to develop the acoustic computer model, which enables to simulate propagation of ultrasound in different media and to calculate the signals reflected by the objects, having a complicated structure. Several modelling approaches for approximation of objects through the series of different facets (rectangular, circular and triangular) are available [1-3].

Different shapes can be described by subdividing the object into a large number of rectangular facets [1]. The facets are described by centre of the facet, the outward normal to the facet, the area of the facet and the length vectors of the two sides of the facet. Scattering from all illuminated facets are summed coherently to get the net scattered pressure [1].

3D objects can be represented by densely packed surfaces of facets that are small compared to the wavelength [2]. The coordinates and the surface normal of the facets are computed and stored. To model 3D objects basic shapes like the sphere, cylinder and cone are superimposed. The basic building block of these shapes is the circle. The object surface is constructed by stacking circles with an appropriate radius one behind other. The insonified facets are computed by selecting facets whose surface normal have a positive component towards the sonar view point. It is assumed that the energy propagates along straight ray paths [2].

The surface of the 3D object also can be represented by plane triangular facets [3]. Each facet in the three-dimensional space is represented by its vertex points and the unit surface normal vector pointing out of the body [3].

For the approximation of inspected objects plane triangular facets were selected, because in a widely used CAD model all surfaces are given in terms of triangles. Also the plane triangular facet is suited for approximation of all types of surfaces because of its co-planar property [3].

The main objective of the developed acoustic model is to calculate ultrasonic signals reflected by the components of a complicated geometry, approximated by triangles, taking into account position, orientation and parameters of the ultrasonic transmitters and receivers. Ultrasonic transducers in most cases operate in a far field zone. In this zone the diffraction effects caused by the geometry of the transducer are very low and can be neglected. Therefore for the selected space point the pulse response of the transducer can be simplified up to two parameters: the signal delay time and the signal amplitude. The reflected signal according to this approach is calculated as a sum of the signals reflected by elementary segments of the reflecting surface - a part of the triangle which is in the intersection zone of the directivity patterns of the transmitter and the receiver.

Main steps of the proposed method

In the used model it is assumed that a triangle is arbitrally oriented in space (Fig. 1). The calculations in 3D space are very complicated and require a lot of computer resources, therefore the triangle plane is rotated in a space in such a way, that after the turn the triangle would be located in one plane (z=0) and all calculations could be performed in 2D space. Then the amount of the data is reduced, and, consequently, the speed of calculations is increased a lot. The transmitter and receiver are rotated also – that means, that the position of transmitter and receiver according to the triangle plane remains unchanged, only all together are rotated in such a way that the triangle would be located in one plane.

The modelling of 3D reflections from triangles using the Huygens approach consists of the following steps:

- The triangle is moved to the origin of the coordinate system;
- The triangle is rotated in such a way, that after the rotation the triangle would be located in one plane (z=0);
- The transmitter and receiver are translated and rotated in the same way in order to keep the position of the transducers unchanged with respect to the triangle plane;
- The zone of the triangle which is completely in the intersection zone of the directivity patterns of the transmitter and receiver is found;
- This triangle zone is divided into elementary segments with a step, which is smaller than the half of the wavelength;

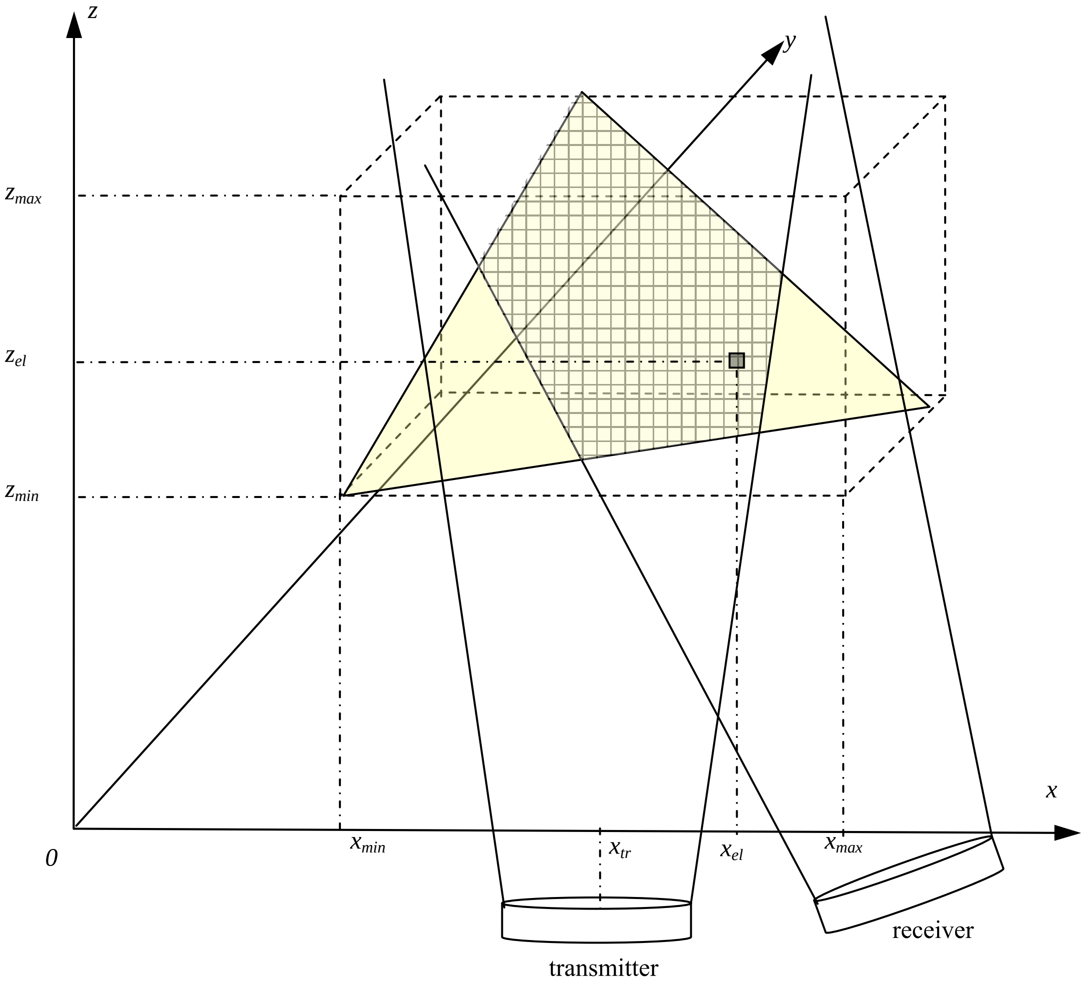


Fig. 1. **Geometry of the problem**

- The distance between these elementary segments and the transducer centre is calculated;
- The amplitude taking into account the directivity pattern, the distance and the delay time of the signal propagating from the transmitter to each of the elementary segments is calculated;
- Each of elementary segments is assumed to be a new source of ultrasonic waves and the distances from them to the receiver are calculated;
- The received signal is calculated using integration and convolution.

Calculation of 3D reflections form triangles

Definition of a triangle position in space

For a definition of a triangle position in space the coefficients of the plane, where a triangle is located, are determined:

$$\begin{aligned} A_{tr} &= (y_2 - y_1)\,(z_3 - z_1) - (y_3 - y_1)\,(z_2 - z_1), \\ B_{tr} &= (z_2 - z_1)\,(x_3 - x_1) - (z_3 - z_1)\,(x_2 - x_1), \\ C_{tr} &= (x_2 - x_1)\,(y_3 - y_1) - (x_3 - x_1)\,(y_2 - y_1), \\ D_{tr} &= -(A_{tr} \cdot x_1 + B_{tr} \cdot y_1 + C_{tr} \cdot z_1)\,. \end{aligned} \tag{1}$$

where x_1, y_1, z_1 are the coordinates of the first vertex of the triangle, x_2, y_2, z_2 are the coordinates of the second vertex of the triangle, x_3, y_3, z_3 are the coordinates of the third vertex of the triangle, A_{tr}, B_{tr}, C_{tr}, D_{tr} are the coefficients of the triangle plane, which can be described by the following equation:

$$A_{tr}x + B_{tr}y + C_{tr}z + D_{tr} = 0\,. \tag{2}$$

The angle of the triangle plane with respect to the z=0 plane has to be found. The angle between two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ can be determined using the following equation:

$$\alpha_{pl} = \arccos\left(\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^{\,2} + B_1^{\,2} + C_1^{\,2}}\sqrt{A_2^{\,2} + B_2^{\,2} + C_2^{\,2}}}\right). \tag{3}$$

In this case the coefficients of the triangle plane are $A_1 = A_{tr}$, $B_1 = B_{tr}$, $C_1 = C_{tr}$, $D_1 = D_{tr}$ and the coefficients of the z=0 plane are $A_2 = 0$, $B_2 = 0$, $C_2 = 1$, $D_2 = 0$. So, the triangle plane angle with respect to the z=0 plane is given by:

$$\alpha_{pl} = \arccos\left(\frac{C_{tr}}{\sqrt{A_{tr}^{\,2} + B_{tr}^{\,2} + C_{tr}^{\,2}}}\right). \tag{4}$$

Translation of the triangle

The first vertex of the triangle is moved to the origin (translation) of the coordinate system. For the translation a 4x4matrix is used. The position vector $\mathbf{p_1}$ is composed as follows (with x_1, y_1, z_1 corresponding to the position of the first vertex of the triangle in 3D space):

$$\vec{\mathbf{p}}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}. \tag{5}$$

The translation matrix $\mathbf{T}$ is as follows (dx, dy, dz are the translation distances along each axis):

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

The result – the translation of the first vertex of the triangle to the origin is given by:

$$\begin{bmatrix} x_{1m} \\ y_{1m} \\ z_{1m} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}. \tag{7}$$

Other vertexes of the triangle are translated also:

$$\begin{bmatrix} x_{nm} \\ y_{nm} \\ z_{nm} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}, \tag{8}$$

where x_n, y_n, z_n are the coordinates of the n-th vertex of the triangle in 3D space, x_{nm}, y_{nm}, z_{nm} are the coordinates of the n-th vertex of the triangle in 3D space after translation.

Rotation of a triangle

The plane of the triangle is rotated around the line, which is on the intersection of the triangle plane and z=0 plane. First of all the directive coefficients of the line on the intersection of the triangle plane and z=0 plane have to be found. The equation of the line in 3D can be written as the intersection of the two planes:

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0, \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}. \tag{9}$$

The directive coefficients of this line can be determined using the following equations:

$$\begin{aligned} l &= B_1 C_2 - B_2 C_1, \\ m &= A_2 C_1 - A_1 C_2, \\ n &= A_1 B_2 - A_2 B_1. \end{aligned} \tag{10}$$

In this case the coefficients of the triangle plane are $A_1 = A_{tr}$, $B_1 = B_{tr}$, $C_1 = C_{tr}$, $D_1 = D_{tr}$ and the coefficients of the z=0 plane are $A_2 = 0$, $B_2 = 0$, $C_2 = 1$, $D_2 = 0$. So, the directive coefficients of the line on the intersection of the triangle plane and z=0 plane are:

$$\begin{aligned} l &= B_{tr}, \\ m &= -A_{tr}, \\ n &= 0. \end{aligned} \tag{11}$$

The unit vector of the same line can be expressed as:

$$\begin{aligned} u_x &= \frac{l}{\sqrt{l^2 + m^2 + n^2}}, \\ u_y &= \frac{m}{\sqrt{l^2 + m^2 + n^2}}, \\ u_z &= \frac{n}{\sqrt{l^2 + m^2 + n^2}}. \end{aligned} \tag{12}$$

The rotation matrix is given by:

$$\mathbf{R} = \begin{bmatrix} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \\ a_3 & b_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

where

$$\begin{aligned} a_1 &= 1 + \left(1 - \cos(\alpha_{pl})\right) \left(u_x^2 - 1\right), \\ a_2 &= \left(1 - \cos(\alpha_{pl})\right) u_x u_y + u_z \sin(\alpha_{pl}), \\ a_3 &= \left(1 - \cos(\alpha_{pl})\right) u_x u_z - u_y \sin(\alpha_{pl}), \\ b_1 &= \left(1 - \cos(\alpha_{pl})\right) u_x u_y - u_z \sin(\alpha_{pl}), \\ b_2 &= 1 + \left(1 - \cos(\alpha_{pl})\right) \left(u_y^2 - 1\right), \\ b_3 &= \left(1 - \cos(\alpha_{pl})\right) u_y u_z + u_x \sin(\alpha_{pl}), \\ c_1 &= \left(1 - \cos(\alpha_{pl})\right) u_x u_z + u_y \sin(\alpha_{pl}), \\ c_2 &= \left(1 - \cos(\alpha_{pl})\right) u_y u_z + u_x \sin(\alpha_{pl}), \\ c_3 &= 1 + \left(1 - \cos(\alpha_{pl})\right) \left(u_z^2 - 1\right). \end{aligned}$$

The rotation matrix is multiplied by the position vector, which in this case is given by the coordinates of the n-th triangle vertex:

$$\vec{\mathbf{p}}_n = \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}. \tag{14}$$

Multiplication of matrices yields:

$$\begin{bmatrix} x_{rn} \\ y_{rn} \\ z_{rn} \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \\ a_3 & b_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}. \tag{15}$$

The coordinates of the vertexes of the rotated triangle are as follows:

$$\begin{aligned} x_r &= a_1 x + b_1 y + c_1 z, \\ y_r &= a_2 x + b_2 y + c_2 z, \\ z_r &= a_3 x + b_3 y + c_3 z. \end{aligned} \tag{16}$$

The coefficients of the rotated triangle plane are given by:

$$\begin{aligned} A_{rtr} &= \left(y_{r2} - y_{r1}\right) \left(z_{r3} - z_{r1}\right) - \left(y_{r3} - y_{r1}\right) \left(z_{r2} - z_{r1}\right), \\ B_{rtr} &= \left(z_{r2} - z_{r1}\right) \left(x_{r3} - x_{r1}\right) - \left(z_{r3} - z_{r1}\right) \left(x_{r2} - x_{r1}\right), \\ C_{rtr} &= \left(x_{r2} - x_{r1}\right) \left(y_{r3} - y_{r1}\right) - \left(x_{r3} - x_{r1}\right) \left(y_{r2} - y_{r1}\right), \\ D_{rtr} &= -\left(A_{rtr} \cdot x_{r1} + B_{rtr} \cdot y_{r1} + C_{rtr} \cdot z_{r1}\right). \end{aligned} \tag{17}$$

where x_{r1}, y_{r1}, z_{r1} are the coordinates of the first vertex of the rotated triangle, x_{r2}, y_{r2}, z_{r2} are the coordinates of the second vertex of the rotated triangle, x_{r3}, y_{r3}, z_{r3} are the coordinates of the third vertex of the rotated triangle and A_{rtr}, B_{rtr}, C_{rtr}, D_{rtr} are the coefficients of the rotated triangle plane, which can be described by the following equation:

$$A_{rtr}x + B_{rtr}y + C_{rtr}z + D_{rtr} = 0 . \qquad (18)$$

Division of the triangle into elementary segments

For each triangle 2D area, in which the given triangle is placed, has to be determined. This 2D area is divided into the elements with a step, which is less then the half wavelength. First of all the approximate step is calculated:

$$\Delta d = \lambda / 2 . \qquad (19)$$

Then the number of elements in x and y directions is calculated:

$$N_x = \frac{x_{\max} - x_{\min}}{\Delta d} + 1,$$

$$N_y = \frac{y_{\max} - y_{\min}}{\Delta d} + 1 , \qquad (20)$$

where x_{max}, x_{min}, y_{max}, y_{min} define the 2D area, where triangle is located.

Translation and rotation of the transducer

The transducers (transmitters and receivers) are translated at the same distance as the triangle in order to keep the position of the transducer according to the triangle plane unchanged:

$$\begin{bmatrix} x_{tm} \\ y_{tm} \\ z_{tm} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_t \\ y_t \\ z_t \\ 1 \end{bmatrix}, \qquad (21)$$

where x_t, y_t, z_t correspond to the position of the transducer centre in 3D space, x_{tm}, y_{tm}, z_{tm} correspond to the position of the translated transducer centre in 3D space.

Then the coordinates of the transducers are calculated in the rotated coordinate system. The same rotation matrix as before is used. The rotation matrix is multiplied by the position vector, which in this case is given by the coordinates of the transducer centre:

$$\vec{\mathbf{p}}_t = \begin{bmatrix} x_t \\ y_t \\ z_t \\ 1 \end{bmatrix}. \qquad (22)$$

The coefficients of the rotated transducer plane are determined rotating the normal vectors of the transducers:

$$\vec{\mathbf{p}}_{tr} = \begin{bmatrix} n_{tx} \\ n_{ty} \\ n_{tz} \\ 0 \end{bmatrix}. \qquad (23)$$

The coefficients of the rotated transducer plane are as follows:

$$\begin{aligned} A_{rt} &= n_{txr}, \\ B_{rt} &= n_{tyr}, \\ C_{rt} &= n_{tzr}, \\ D_{rt} &= -\left(A_{rt} \cdot x_{rt} + B_{rt} \cdot y_{rt} + C_{rt} \cdot z_{rt}\right), \end{aligned} \qquad (24)$$

where x_{rt}, y_{rt}, z_{rt} are the coordinates of the rotated transducer centre and A_{rt}, B_{rt}, C_{rt}, D_{rt} are the coefficients of the rotated transducer plane, which can be described by the following equation:

$$A_{rt}x + B_{rt}y + C_{rt}z + D_{rt} = 0 . \qquad (25)$$

Determination of elements of a triangle covered by the directivity pattern of a transducer

Now it has to be found which points of the triangle plane are covered by the directivity pattern of each transducer. First, the distance from the points in the triangle plane to the centre of the transducer is calculated

$$d_1 = \sqrt{\left(x_{rt} - x_{el}\right)^2 + \left(y_{rt} - y_{el}\right)^2 + \left(z_{rt} - z_{el}\right)^2} , \qquad (26)$$

where x_{rt}, y_{rt}, z_{rt} are coordinates of the transducer centre, x_{el}, y_{el}, z_{el} are the coordinates of the elementary segment in the triangle plane (Fig. 1).

Then the distance from the plane of the triangle to the plane of the transducer as a normal is calculated:

$$d_2 = \frac{A_{rt}x + B_{rt}x + C_{rt}x + D_{rt}}{\sqrt{A_{rt}^2 + B_{rt}^2 + C_{rt}^2}} \qquad (27)$$

where A_{rt}, B_{rt}, C_{rt}, D_{rt} are the coefficients of the rotated transducer plane $A_{rt}x + B_{rt}y + C_{rt}z + D_{rt} = 0$.

The distance from the transducer centre to the point, where the normal from the triangle plane hits the transducer plane is calculated:

$$d_3 = \sqrt{\left|d_1^2 - d_2^2\right|} . \qquad (28)$$

The signal amplitude in each point of the triangle plane according to the directivity pattern of the transducer is found:

$$A = \left(\exp\left(K_{dir} \cdot \alpha_a^2\right)\right)^2 , \qquad (29)$$

where $K_{dir} = \log(0.5)/\alpha_r^2$; $\alpha_a = \arctan\left(d_3/d_2\right)$ is the angle from the transducer axis, α_r is the limiting angle of the transducer.

The signal propagation time from the transmitter to the elementary segment in the triangle and to the receiver is calculated, because all elements are assumed to be the sources of ultrasonic waves:

$$t = \frac{d_{et} + d_{er}}{c} , \qquad (30)$$

where d_{et} is the distance from the elementary segment in the triangle plane to the centre of the transmitter, d_{er} is the distance from the elementary segment in the triangle plane to the centre of the receiver, c is the ultrasound velocity.

Calculation of the received signals

As it was described above, the triangle is decomposed into elementary segments and the delay time and the amplitude of the signals reflected by these segments are calculated. So, the pulse response of the object is presented as the set of pairs $\mathbf{H_{ob}} = \left\{\left(A_1, t_1\right), \left(A_2, t_2\right), \ldots, \left(A_{N_e}, t_{N_e}\right)\right\}$, where A_k and t_k are the amplitude and the delay time of the signal reflected by k-th segment correspondingly, $k = 1 \ldots N_e$, N_e is the total number of elementary segments.

According to the Huygens's principle the total reflected signal can be expressed as the sum of reflections from the elementary segments as

$$u(t) = \sum_{k=1}^{N_e} u_t(t) \otimes h_k(t), \qquad (31)$$

where the $u_t(t)$ is the transmitted ultrasonic signal $h_k(t)$ is the pulse response of k-th elementary segment, $\otimes$ denotes convolution.

The transmitted signal, which has a shape of a high frequency pulse with the Gaussian envelope, was approximated by:

$$u_t(t) = \mathrm{e}^{a(t-b)^2} \sin(2\pi f t), \qquad (32)$$

where $a = k_a f \sqrt{\dfrac{-2\ln 0.1}{p_s}}$, $b = \dfrac{2p_s}{3f}$, p_s is the number of periods, k_a is the asymmetry factor, f is the frequency.

Modelling of 3D reflections from triangle at different angles

In order to test the developed model, simulation of ultrasonic signals reflected by a triangle immersed in water was carried out. The geometry of the triangle is given in Fig. 2. The simulation was performed using the single 5 MHz ultrasonic transducer, operating in a pulse-echo mode. The transducer was at 300 mm from the object. The transducer was consequently shifted in a plane (z=300 mm), and the simulated reflected signals in the time domain were used to construct a C-scan type image of the triangle. The scanning step was 0.5 mm.

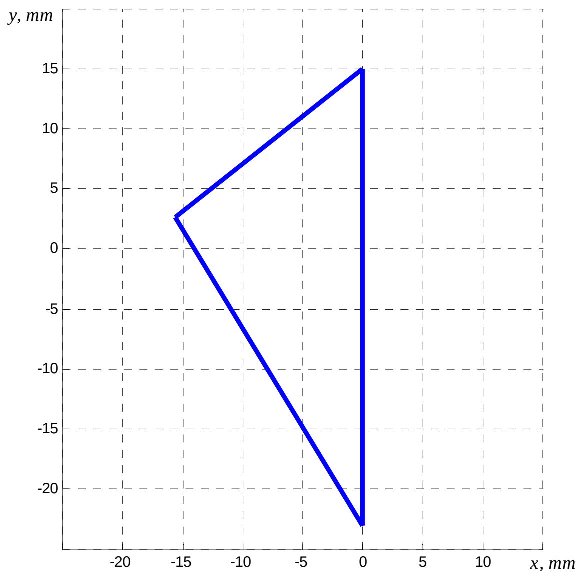
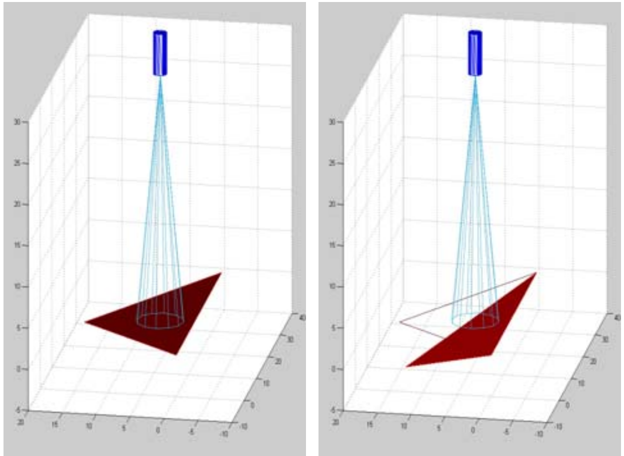


Fig. 2. **Geometry of the simulated triangle**

Modelling of the 3D reflections form the triangle was performed in the case when an ultrasonic beam is reflected by a planar surface and by triangles, reflecting surfaces of which are inclined with respect to the symmetry axis of the directivity pattern, therefore the signals which are picked up are not specularly reflected by the object. For this purpose the single triangle was rotated around the longest leg (Fig.3).

The simulated ultrasonic images of the triangle reflector obtained under different orientation angles are presented in Fig. 4 - 9. It is necessary to take into account that the true value of the maximal amplitude in each image differs essentially. In all images the presented amplitudes are normalized with respect to the maximal amplitude.



a b

Fig. 3. **Rotation of the single triangle around the longest leg a – specular reflection, b – inclined reflection**

The following conclusions can be made from the presented images:

- A triangle reflector as such can be recognized only under angles close to perpendicular to the triangle surface;
- The edges of the triangle can be seen when they are parallel to the transducer surface – the longest leg in Fig.4-9.

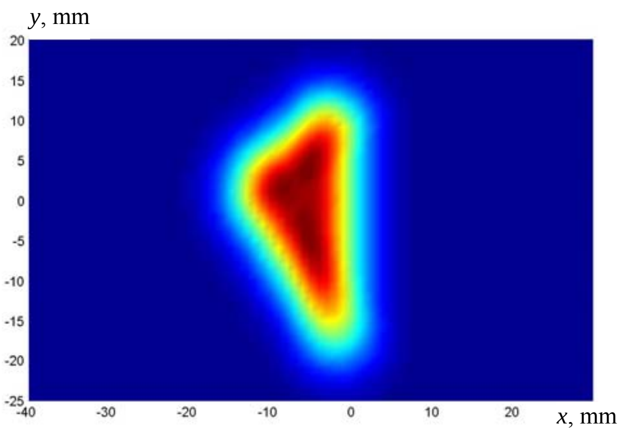


Fig.4. **The simulated C-scan image of the triangle at the angle 0°**

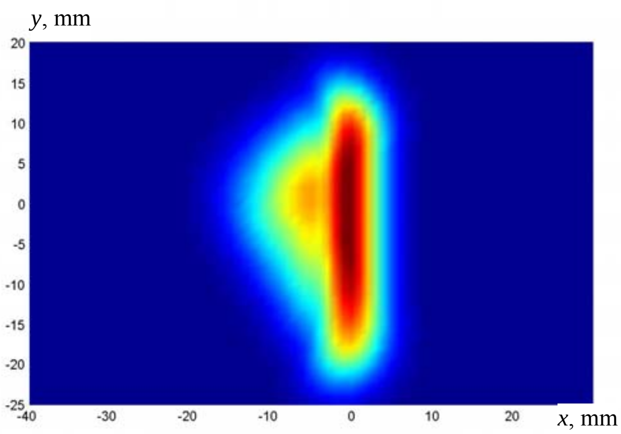


Fig.5. **The simulated C-scan image of the triangle at the angle 1°**

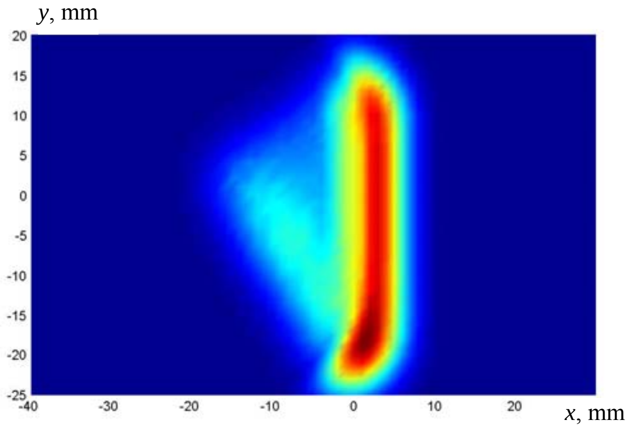


Fig.6. **The simulated C-scan image of the triangle at the angle 2°**

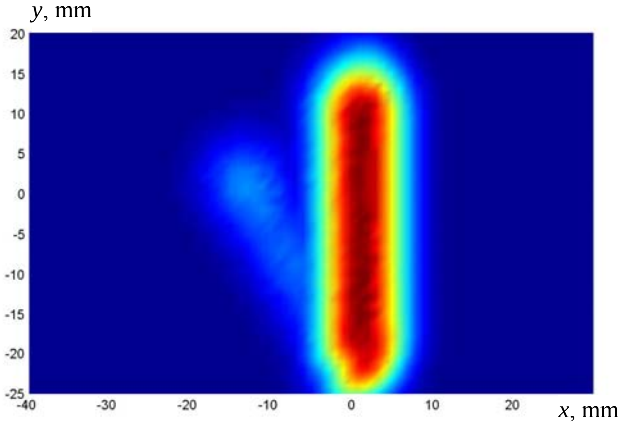


Fig.7. **The simulated C-scan image of the triangle at the angle 3°**

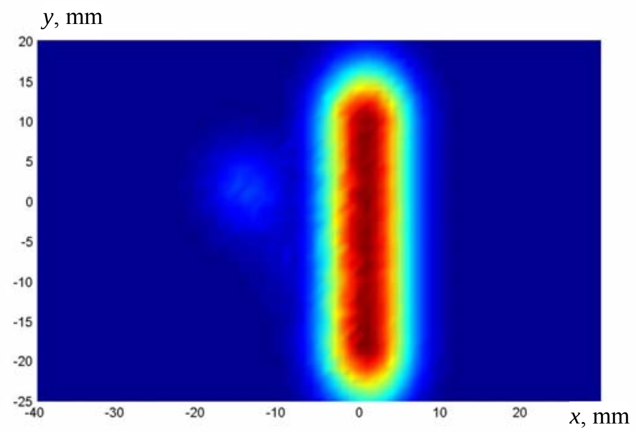


Fig.8. **The simulated C-scan image of the triangle at the angle 4°**

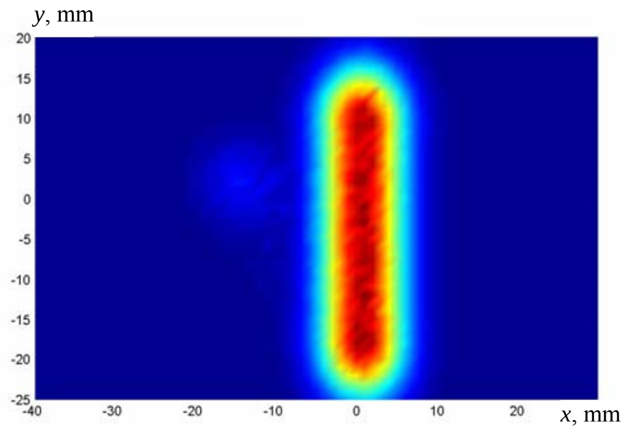


Fig.9. **The simulated C-scan image of the triangle at the angle 5°**

Conclusions

The developed acoustic model enables to calculate the signals reflected by triangles in 3D space using the Huygens approach. The rotation of the triangle in such a way, that after the turn the triangle would be located in one plane allowed to reduce the amount of data and, consequently, to increase the speed of calculations. The simulations of the 3D reflections from triangle at different angles show, that a triangle as a shape can be recognised only in the case of the specular reflection. The edges of the triangle can be seen when they are parallel to the transducer surface.

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E. Jasiūnienė

Atspindžių nuo trikampių modeliavimas trimatėje erdvėje naudojant Hiuigenso principą

Reziumė

Šio darbo tikslas buvo sukurti tokį akustinį kompiuterinį modelį, kuris leistų modeliuoti ultragarso signalo atspindžius nuo bet kaip orientuotų trikampių trimatėje erdvėje. Trikampio pasukimas taip, kad šis atsirastų z=0 plokštumoje, leido sumažinti duomenų kiekį ir kartu padidinti skaičiavimų spartą. Atspindžių nuo trikampio modeliavimas esant įvairiems jo pasukimo kampams parodė, kad trikampio forma gali būti atpažinta, tik kai trikampio plokštuma yra lygiagreti su keitiklio plokštuma. Be to, matomos tik tos trikampio kraštinės, kurios yra lygiagrečios su keitiklio plokštuma.

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