Sound insulation of multi-layer cylindrical structures

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Abstract

The analytical methods for computation of sound insulation of multi-layer cylindrical constructions are formulated in the paper. The relationship between multi-layer and one-layer constructions is specified. With the use of analytical methods described in the paper, sound insulation of one-layer and multi-layer constructions was computed.

The results obtained showed that sound insulation of a one-layer construction of the same thickness was by two orders less in comparison with the sound insulation of multi-layer construction.

Keywords: cylindrical constructions, sound insulation, noise reduction.

Introduction

It has been already known from the literature that sound insulation of the walls of cylindrical surface of the construction (shell) differs from the frequency characteristic of sound insulation of the boards [1 - 4]. Difference is important due to the fact that surfaces of cylindrical shape insulate considerably better the low-frequency sound than the board from the same material. That anomaly of sound insulation focused the attention of scientists to the use of that property for abatement of a low-frequency noise.

At present, the multi-layer constructions from polymer structures became widespread as efficient means for vibro- and sound insulation [5, 6]. Moreover, special attention is devoted to multi-layer polymer cylindrical shells. However, the solution of elastoacoustic problems of dynamics of polymer cylindrical shells is related with great mathematical difficulties. In the present work estimation is made of the sound insulation of a multi-layer polymer cylindrical structure on the basis of the approximated Helmholtz equation with variable coefficients, characterizing the properties of a multi-layer construction.

Theory

We shall study the acoustic field in the area, limited by two infinite coaxial direct circular cylinders ($R_1 \prec r \prec R_2; 0 \le \varphi \le \pi, -\infty \prec z + \infty$). (Fig. 1). Pressure P inside that inhomogeneous area satisfies a wave equation:

$$\frac{\partial^2 p}{\partial t^2} = \frac{c^2}{q} \nabla q \nabla p + c^2 \nabla P_1 \tag{1}$$

where $q = \frac{1}{\rho}$; ρ, c is sound density and velocity in the medium.

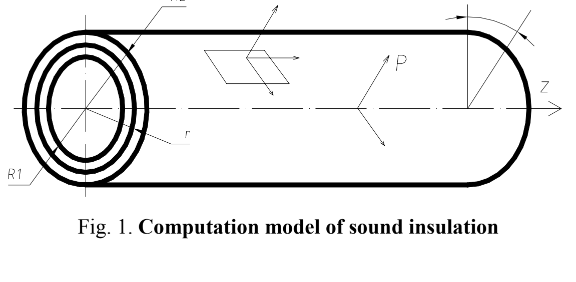

Fig. 1. **Computation model of sound insulation**

We shall search for equation solution in the form

$$P(r, \varphi, t) = u(r, \varphi) \cdot e^{i\omega t} . \tag{2}$$

Substituting Eq.2 into Eq.1, we get

$$\Delta u + k^2 u + \frac{1}{q} \nabla q \cdot u = 0 \tag{3}$$

We assume that $q = q\ (r,\ \varphi)$ is a slowly changing function in the sense that $\left| \frac{\nabla q}{q} \right| \prec\prec 1$.

Let's study the inhomogeneous Helmholtz equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u.}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + k^2(r)u = f(r, \varphi). \tag{4}$$

Assuming that the right side is represented by the Fourier series

$$f(r, \varphi) = \sum_{n=-\infty}^{\infty} f_n(r) e^{-in\varphi} , \tag{5}$$

where

$$f_n(r) = \frac{1}{2\pi} \int_0^{2\pi} f(r, \varphi) \cdot e^{-in\varphi} \cdot d\varphi , \tag{6}$$

We shall search for a solution of Eq. 5 in term of

$$u(r, \varphi) = \sum_{n=-\infty}^{\infty} u_n(r) \cdot e^{in\varphi} , \tag{7}$$

where $u_n(r)$ satisfies the following ordinary differential equation

$$\frac{d^2 u_n(r)}{dr^2} + \frac{du_n(r)}{dr} - \frac{n^2}{r^2} u_n(r) + k^2(r) u_n(r) = f_n(r) . \tag{8}$$

Let us write the general solution of the one-layer equation Eq. 8, in the form

$$u(r, \varphi) = \sum_{n=-\infty}^{+\infty} \left[A_n \cdot u_n^{(1)}(r) + B_n u_n^{(2)}(r) \right] \cdot e^{in\varphi} , \tag{9}$$

where A_n and B_n are arbitrary constants.

The general solution of the inhomogeneous Eq. 8 may be found, for example, by the method of variation of arbitrary constants. Setting the boundary conditions on circles $r = R_1$ and $r = R_2$, it is possible to define the constants A_n and B_n from Eq. 9. Here, the boundary conditions are determined by an external acoustic field. After finding A_n and B_n, Eq. 9 determines fully the acoustic field inside the construction under a study.

As a simplified example of the described methods we shall study a two-layer construction, described on the basis of the Helmholtz equations with piecewise –constant coefficients, characterizing the properties of each of the layers.

Suppose, we have a direct circular cylinder $(0 \le r \prec R_1; 0 \le \varphi \le 2\pi, -\infty < z + \infty)$, filled with the medium with the acoustic resistance $\rho_1 c_1$. It is encircled by two protective cylindrical layers $(R_1 \prec r \prec R_2;\ 0 \le \varphi \le 2\pi; -\infty \prec z \prec +\infty$ and $R_2 \prec r \prec R_3, 0 \le \varphi \le 2\pi, -\infty \prec z \prec +\infty)$ with the acoustic resistances that are equal correspondingly to $\rho_2 c_2$ and $\rho_3 c_3$. Let us write the potential of velocities $W_j(\rho, \varphi, z, t)$, $(j = 1 = 1,2,3)$ in the indicated three media. In each of them it satisfies the wave equation:

$$\frac{\partial^2 W_j}{\partial t^2} = c_j^2 \Delta(r, \varphi, z) W_j . \qquad (10)$$

Let us set on the external surface *(r=R₃)* of the construction under study the normal velocity component

$$\frac{\partial W_3}{\partial r} /_{r=R_3} = e^{i\omega t} \sum_{n=-\infty}^{\infty} u_e^{in\varphi}; \quad \begin{pmatrix} \omega = const \\ v = const \\ n = 0, \pm 1, \pm 2 \end{pmatrix}. \qquad (11)$$

Then the potential of velocities does not depend on z (the planar problem)

$$W_j = e^{i\omega t} u_j(r, \varphi) \qquad (12)$$

and we shall search for its peak value in the form

$$u_j(r, \varphi) = \sum_{n=-\infty}^{+\infty} u_{jn}(r) e^{in\varphi} . \qquad (13)$$

The functions $u_{j(r,\varphi)}$ satisfy in the corresponding areas the Helmholtz equation with the wave numbers $k_j = \frac{\omega}{c_j}$

$$\Delta u_j(r, \varphi) + k_j^2 u_j(r, \varphi) = \sum_{n=-\infty}^{+\infty} \left[u''_{jn}(r) + \frac{1}{2} u'_{jn}(r) + (k^2 - \frac{n^2}{r^2}) + u_{jn}(r) \right] e^{in\varphi} = 0 \qquad (14)$$

On the boundary circles the following conditions should be met

$$\rho_1 u_{in}(R_1) = \rho_2 u_{2n}(R_1); \rho_2 u_{2n}(R) = \rho_3 u_{3n}(R_2);$$
$$\frac{\partial u_{1n}}{\partial r} /_{r=R_1} = \frac{\partial u_{2n}}{\partial r} /_{r=R_1}; \ \frac{\partial u_{\cdot 2n}}{\partial r} /_{r=R_2} = \frac{\partial u_{3n}}{\partial r} /_{r=R_2} \qquad (15)$$
$$\frac{\partial u_{3n}}{\partial r} /_{r=R_3} = V_n, \ n = 0, \pm 1, \pm 2 ...$$

The solution of Eq. 14 for each of harmonics is written in the cylindrical functions

$$u_{jn}(r) = A_{jn} J_n(k_j r) + B_{jn} Y_n(k_j r), (j = 1,2,3). \qquad (16)$$

Substituting Eq.16 into the boundary conditions Eq.15, we'll get the system of algebraic equations with respect to the constants A_{jn} and B_{jn}. Solving it, we have for the amplitude of the potential in the small circle $(0 \le r \le R_1)$

$$u_n(r, \varphi) = \frac{D}{\sqrt{2}} R_1 \sum_{n=-\infty}^{-\infty} \frac{V_n}{\Delta n} J_n(k, r) e^{in\varphi} \qquad (17)$$

where it is defined $\left(\frac{2}{\pi \mu_{21}}\right)^2 \frac{1}{\mu_{32}} \frac{\rho_3}{\rho_2} = \frac{D}{\sqrt{2}}, \mu_{\upsilon j} = k_\nu r_j$, $(\upsilon, 1 = 1,2,3)$. Now let us suppose that the protection rings $R_1 \prec r \prec R_2, R_2 \prec r \prec R_3, 0 \le \varphi \prec 2\pi$, separating the small circle $0 \le r \prec R_1$, $0 \le \varphi \prec 2\pi$ from the external acoustic field are absent and the circle $r=R_1$ gets excited by the external acoustic field with the radial velocity

$$e^{i\omega t} \sum_{n=-\infty}^{+\infty} v_n e^{in\gamma} .$$

Let $u_1^* = u_1^*(r, \varphi)$ is the amplitude of the potential of the velocities of the acoustic field of the small circle. Then $u_1^*(r, \varphi)$ is the solution of the following Neumann problem:

$$\Delta u_1^* + k_1^2 u_1^* = 0, \ \frac{\partial u_1^*}{\partial r} /_{r=R_1} = \sum_{n=-\infty}^{\infty} v_n e^{in\varphi} . \qquad (18)$$

Let us suppose that $J_n^1(k_1 R_1) \ne 0 (n = 0, \pm 1, \pm 2, ...)$, then

$$u_1^*(r, \varphi) = \frac{R_1}{\mu_{11}} \sum_{n=-\infty}^{\infty} V_n \frac{J_n(k_1 r)}{J_n^1(\mu_{11})} e^{in\varphi} . \qquad (19)$$

Let us compute the mean square pressure per unit of the surface of the small circle („norm“)

$$(P_1)_{med} = \frac{1}{\pi R_1^2} \| \rho_1 u_1(r, \varphi) \| = \frac{1}{\sqrt{2} \pi R_1^2} D R_1 \rho_1 \left\{ \int_0^{R_1} r dr \int_0^{2\pi} \left(\sum_{n=-\infty}^{+\infty} \frac{v_n}{\Delta n} J_n(k, r) e^{in\varphi} \right)^2 d\varphi \right\}^{\frac{1}{2}} = \qquad (20)$$
$$= \frac{D \rho_1}{\sqrt{\pi} \mu_{11}} \left\{ \sum_{n=-\infty}^{+\infty} \frac{v_n^2}{\Delta n^2} \int_0^{\mu_{11}} x J_n^2(x) dx \right\}^{\frac{1}{2}}, (x = k, r).$$

The analogous value for the unprotected circle has the form

$$(P_1^*)_{med} = \frac{1}{\pi R_1^2} \| P_1^*(r, \varphi) \| = \frac{\sqrt{2}}{\sqrt{\pi}} \frac{\rho_1}{\mu_n^2} \left\{ \sum_{n=-\infty}^{\infty} \frac{v_n^2}{J_n^{12}(\mu_{11})} \int_0^{\mu_{11}} x J_n^2(x) dx \right\}^{\frac{1}{2}} . \qquad (21)$$

For verification of the proposed methods for computation of the sound insulation of the construction described, a study was made of the practically used thermal sound insulation material.

The computations performed showed that from the selected materials in the example, the highest effect was received after investigating the cylindrical layered construction.

Computing the construction from the uniform material but with the same thickness of walls as from layered material (from different materials) we get the sound insulation which was by two orders less.

Conclusion

For improving the sound insulation of the walls of cylindrical constructions, it is recommended to apply a two-layer or multi-layer construction from different materials.

The obtained interrelations of the layered cylindrical construction may be used in computing their sound permeability through the walls of the cylindrical shell. The composite layers of the construction may be not only the solid bodies (materials), but also fluid bodies (fluid materials). As an example, we may indicate the following materials: rubber with various specifications; some soft plastic and polymer materials and mastics (caouchouc, glycerin), etc.

Thus, the elaborated methods make it possible to estimate the protective acoustic properties of a multi-layer cylindrical structure at the preset external and internal radius of construction and, on the contrary, at the required protective characteristics to find the necessary layered material and the order of their arrangement.

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Daugiasluoksnės cilindrinės konstrukcijos garso izoliacija

Reziumė

Suformuluota analitinė metodika daugiasluoksnių cilindrinių konstrukcijų garso izoliacijai apskaičiuoti. Nustatytas ryšys tarp daugiasluoksnės ir vienasluoksnės konstrukcijos. Naudojantis minėta analitine metodika apskaičiuota vienasluoksnių ir daugiasluoksnių cilindrinių konstrukcijų garso izoliacija. Tyrimų rezultatai parodė, kad tokio pat storio vienasluoksnės konstrukcijos garso izoliacija dviem eilėmis mažesnė už daugiasluoksnės konstrukcijos garso izoliaciją.

Pateikta spaudai 2007 12 10