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Investigation of possible minimization of number of measurement points in 2D acoustic field reconstruction*

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Abstract

This study investigates the dependency of accuracy of 2D field reconstruction on the number of points of measurement. To create a mathematical model, a regression equation was used and to evaluate the accuracy of reconstruction the sum of residual squares was used. The adequacy of the final model was evaluated by the Fisher criterion. It was shown that the accuracy of the mathematical model logarithmically depends on the number of points of measurement, whose results were used to form the mathematical model.
Keywords: sound pressure level, 2D acoustic field reconstruction, regression analysis

Introduction

To fulfill the scientific or industrial needs it is necessary to make measurements of sound pressure level. This is a demanding task, especially when acoustic parameters must be taken at different points in a room. However, it is not said how many points must be taken to measure the sound pressure level in order to reconstruct acoustic field at certain area or space. It is also not clear what to do when the measurement points cannot be reached or what number of points one should have to get a suitable accuracy. This study analyses this problem as the task of plane interpolation, using linear regression and the method of the least squares (MLS) in order to find the coefficients of regression equation.

Measurements

In the accredited KTU laboratory we took measurements of sound pressure level (SPL) at *xy* plane at 19 points of measurement (PM). The measurements (see Fig. 1) were taken at the height of 1.55 m. according to the requirements [1].

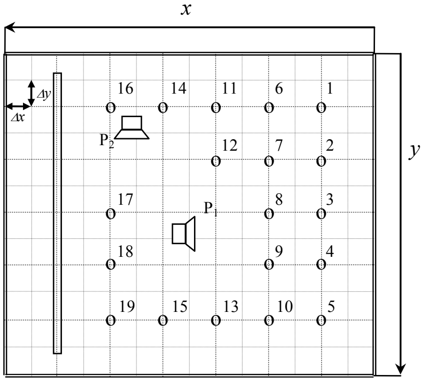

Fig.1. ***xy* plane of measurement (*Δx*= *Δy*=0.5 m). o – is locations of points of measurements**

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The SPL was measured using the Brüel&Kjær precise gauge of acoustic signals INVESTIGATOR. The data were processed by the Noise Explorer (type 7815, ver. 4.1), MAPLE 12 Student edition and the MS Excel software.

Each measurement lasted 2s and was repeated 30 times. During the experiment the acoustic noise was generated by two noise sources, whose power was accordingly P_1=90.57dB and P_2=86.87dB. During the measurement we have got the values of sound pressure at 19 measurement points.

Using these SPL values, a two-dimensional mathematical model (MM) of distribution of SPL at *xy* plane was formed, where *x* and *y* (coordinates) are two independent variables. As the basis of this MM, the regression equation was analyzed (until degree 3) [3]. The parameters of this equation were calculated using the method of the least squares with the experimental data. The adequacy of the zero hypothesis of the mathematical method was checked using F-test (with α=0.05 ir α=0.001 significant level)[4]. The model is adequate when:

$$F_{calc} \le F_{table}\,, \tag{1}$$

where F_{calc} is the Fisher number calculated and described on (2), F_{table} is taken from the *F*-table according to the number of degree of freedom φ_1 and φ_2 and significant level α.

$$F_{calc} = \frac{S_{ad}^2}{S^2\{y\}} \tag{2}$$

where S_{ad}^2 is the adequacy dispersion and $S^2\{y\}$ is the experimental dispersion that are calculated in the following way:

$$S^2\{y\} = \frac{SSE}{\varphi_2} \tag{3}$$

where *SSE* is the the smallest sum of the squared errors (*SSE*):

$$SSE = \sum_{u=1}^{N}\sum_{j=1}^{n}\left(y_{uj} - y_u\right)^2, \tag{4}$$

$$\varphi_2 = N(n-1) \tag{5}$$

where φ_2 is the number of degrees of freedom, *N* is the number of points of measurement, *n* is the number of measurements at each measurement point, y_{uj} is the value of the sound pressure level at the N point of measurement during the n measurement, y_u is the mean of the sound pressure level at the N point of measurement,

$$\sigma_{SPL} = \frac{SSE}{n \cdot N}, \qquad (6)$$

where σ_{gsl} is the variance of the sound pressure level at one point of measurement.

The smallest sum of squared errors (*SSE*) obtained by calculating all variants [2] is shown in Table 1. *SSE* is calculated not only at the points used for forming the mathematical model (if the number of PM is fewer than 19), but at all 19 points.

Table 1. **Experimentally obtained *SSE* and calculated $\sigma^2_{SPL0.05}$ and $\sigma^2_{SPL0.001}$**

Number of PM used to form the MM	Minimal *SSE*	$\sigma^2_{SPL0.05}$	$\sigma^2_{SPL0.001}$
19	0.2006	0.02468	0.01177
18	0.2093	0.02572	0.01227
17	0.2299	0.02825	0.01350
16	0.2885	0.03545	0.01692
15	0.4097	0.05040	0.02402
14	1.199	0.08173	0.04682
13	1.305	0.08892	0.05103
12	1.380	0.09404	0.05391
11	1.406	0.0958	0.05494
10	2.373	0.1241	0.07612
9	2.380	0.1245	0.07628
8	2.712	0.1419	0.08702
7	3.911	0.1673	0.1071
6	3.959	0.1694	0.1085
5	4.079	0.1745	0.1119
4	15.09	0.5771	0.3789
3	16.34	0.6459	0.4175
2	40.9	1.488	0.9842
1	52.6	1.814	1.218

The graphs of *SPL* distribution when the mathematical model is formed using 19, 15, 11, 8, 5 and 3 points of measurement (out of 19 possible) are given in Fig.2 - 7:

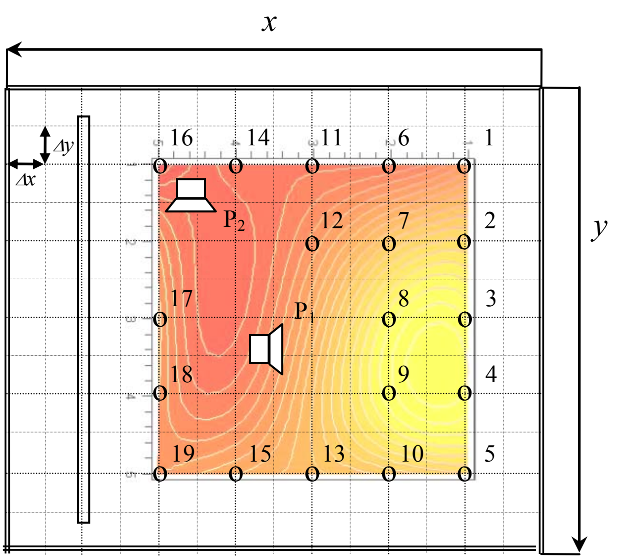

Fig.2. ***xy* plane of measurement with isobars (*Δx*=*Δy*=0.5m) of the reconstructed acoustic field using 19 points of measurement (*SSE*=0.2006)**

Table 2. **The dependence of coefficients of regression equation on the PM number, used for reconstruction**

Number of PM used to form the MM	Number of possible PM and the number of ways of their different location (min *SSE* variant).	Number of coefficients of regression equation used to form MM.
19	1 (1)	15
18	19 (7)	15
17	171 (62)	15
16	969 (622)	15
15	3876 (2020)	15
14	11628 (7859)	11
13	27132 (26440)	11
12	50388 (48120)	11
11	75582 (19196)	11
10	92378 (90611)	8
9	92378 (88949)	8
8	75582 (73347)	8
7	50388 (13416)	5
6	27132 (5438)	5
5	11628 (86)	5
4	3876 (3433)	3
3	969 (286)	3
2	171 (4)	2
1	19 (13)	1

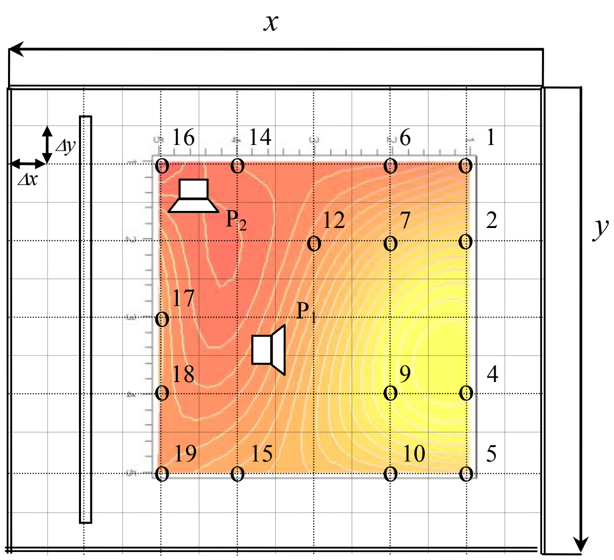

Fig.3. ***xy* plane of measurement with isobars of the reconstructed acoustic field using 15 points of measurement (*SSE*=0.4097)**

For example: reconstruction with 5 points of measurement (out of 19 possible) there are $C^5_{19} = 11628$ possible variants (see Table2) of positioning of these PM at the plane *xy* (placing them at the initial coordinates of 19 PM). The smallest value *SSE* is obtained when there are 5 coefficients of regression equation:

$$SPL = 74.425 + 3.625x - 0.625y + 0.025xy - 0.5x^2. \qquad (7)$$

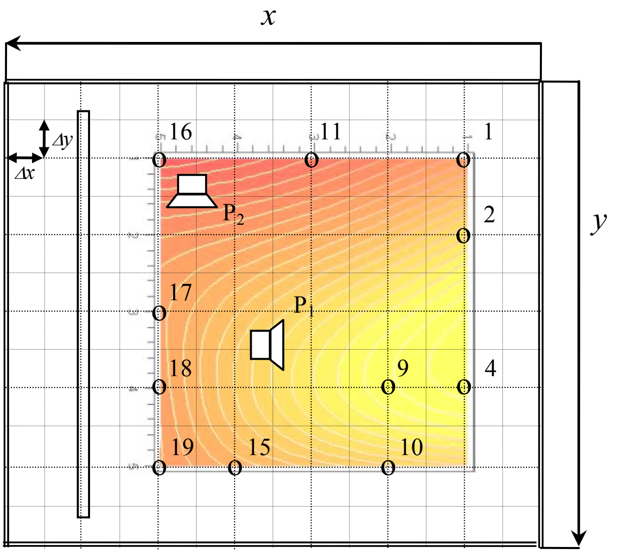

Fig.4. ***xy* plane of measurement with isobars of the reconstructed acoustic field using 11 points of measurement (*SSE*=1.406)**

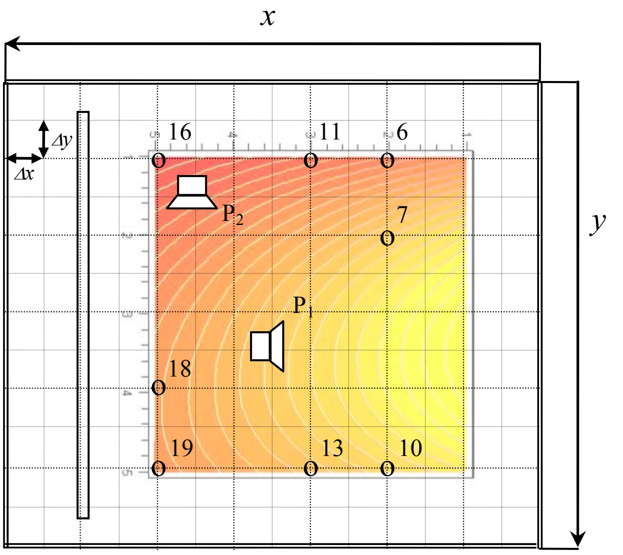

Fig.5. ***xy* plane of measurement with isobars of the reconstructed acoustic field using 8 points of measurement (*SSE*=2.712)**

It should be noticed that the number of coefficients of the regression equation is very important to the adequacy of the mathematical model and the bigger the number, the more accurate the mathematical model is. The investigation conducted by the authors has indicated that when the number is more than 8, the accuracy of the mathematical model saturates. Therefore, in the procedures of the method of smallest squares there were 10 digits used (digits:=10, MAPLE12 [5]). In this investigation, in the measurement procedures we have used 4 significant numbers (while measuring *SPL*). The meaning of the two last columns ($\sigma^2_{SPL0.05}$ *and* $\sigma^2_{SPL0.001}$) of Table1 is this (see Eq.6): when checking the hypothesis of adequacy of the mathematical method, the Fisher criterion is used to compare the adequacy dispersion (S^2_{ad}) and the experimental dispersion $S^2\{y\}$ and the model is adequate when $F_{calc} \leq F_{table}$ [4].

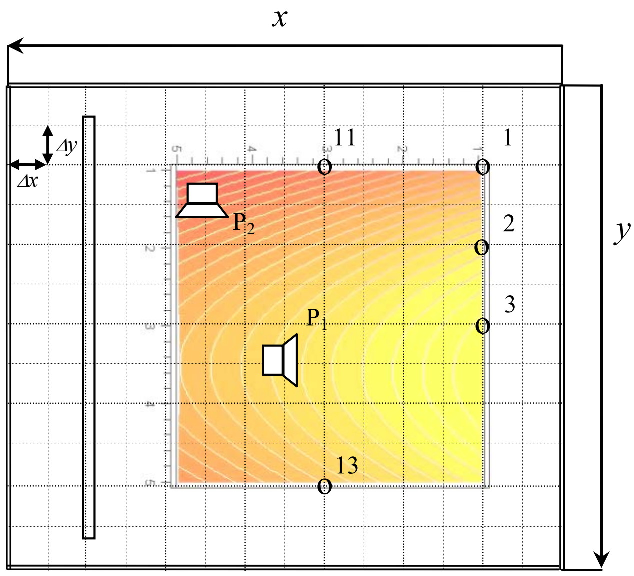

Fig.6. ***xy* plane of measurement with isobars of the reconstructed acoustic field using 5 points of measurement (*SSE*=4.079)**.

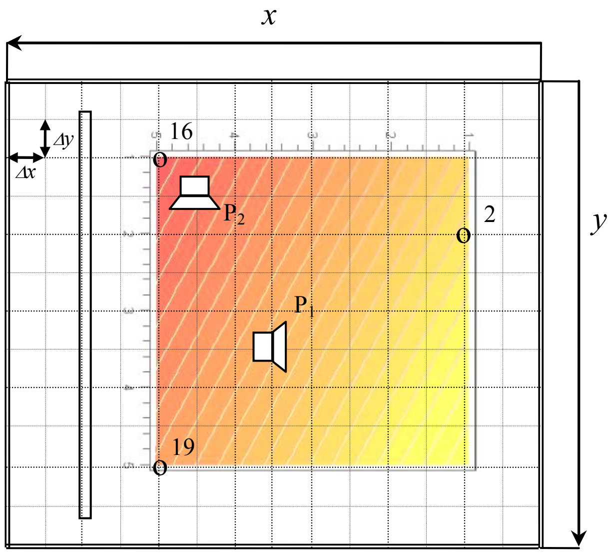

Fig.7. ***xy* plane of measurement with isobars of the reconstructed acoustic field using 3 points of measurement (*SSE*=16.34)**

When *SSE* is divided by the number of PM and the number of measurements at this point we get variance of SPL (σ^2_{SPL}). The variances are calculated for several values of α. This calculation was made under several conditions: the variance is steady and the values of SPL are normally distributed. This is what variance should be like, to make the mathematical model adequate.

Here goes a very important conclusion of this work: the application of this method depends on the character of the measured surroundings (σ^2_{SPL}).

As we can see at Fig. 8, *SSE* increases when logarithmically decreases the number of points of measurement, which is used to form the mathematical model.

The equation, according to the *SSE* dependency on the number of PM (trendline) is:

$$SSE = 40.30e^{-0.294MT} \tag{7}$$

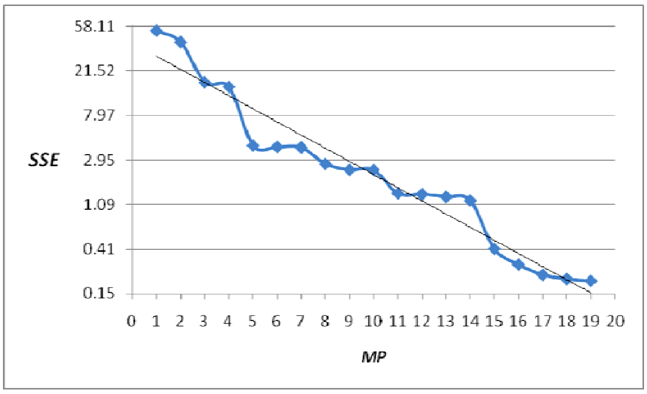

Fig.8. **The dependence of minimal *SSE* on the number of points of measurement (out of 19 possible) used for reconstruction.**

In TSDI Laboratory of Acoustics we did not get the adequate mathematical model, because experimental $\sigma^2{}_{SPL\,\exp}$=0.005275, i.e. when there are 19 MP, when SSE is minimal, $F_{calc} - F_{tble0.05}$=-9.632 and $F_{calc} - F_{table0.001}$=-9.760 (when the MM is adequate, it should be bigger or equal to zero). This is due to the fact that the mode of sound source was strongly static. However equation (7) could be used to get a possible number of MP, which is necessary to form an adequate MM in this experiment. When α=0.05, the MP is 24, when α=0.001, it is 20.

Measurements carried out in other industrial objects show that variance of SPL is possible from 0.05 till 2.00.

Conclusions

According to the results of the investigation of 2D field reconstruction, the following conclusions could be drawn:

1. The sum of squares of residual differences of this mathematical method increases logarithmically, when the number of points of measurement, used for formation of this mathematical method, decreases.
2. The number of points of measurement, needed to form an adequate model, depends on variance of experimental data and required specifications for statistical reliability (α) of mathematical model.

This method could be expanded by analyzing separate components of data spectrum and positioning of these at the reconstructed field. This work does not cover investigation of optimal positioning of measurement points and the influence of measurements uncertainties on quality of reconstruction. However, it indicates what the greatest accuracy of the mathematical method could be obtained, while using regression equation.

Acknowledgements

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Matavimo taškų kiekio 2D akustiniam laukui atkurti minimizavimo galimybių paieška

Reziumė

Tirta 2D akustinių laukų pjūvių atkūrimo tikslumo priklausomybė nuo matavimo taškų kiekio. Matematiniam modeliui kurti naudota regresijos lygtis, o atkūrimo tikslumas įvertintas liekamųjų kvadratų suma (*SSE*) visuose pirminio matavimo taškuose. Gautojo modelio adekvatumo hipotezė patikrinta Fišerio kriterijumi. Nustatyta, kad akustinio lauko atkūrimo matematinio modelio tikslumas logaritmiškai priklauso nuo matavimo taškų kiekio.

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