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Effect of continuous defects to the vibrations of a sheet of paper

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Abstract

The effect of the defects of the modulus of elasticity and of the density to the eigenvalues of bending vibrations of a sheet of paper is analyzed. Graphical representations of the effects of change of the modulus of elasticity and of the density for the first eigenmodes are determined by using the procedure of conjugate smoothing.

By using the procedure of experimental investigations on the basis of the method of projection moire the eigenmodes of the paper excited by vibrations were determined for the paper with defects and without them. The proposed method of optical diagnostics of defects is applicable in the systems of automatic control of packaging materials.

Keywords: paper, bending vibrations, defect, eigenmode, eigenvalue, conjugate approximation, finite elements, conjugate smoothing, projection moiré, experimental setup, diagnostics of defects, automatic control.

1. Introduction

The effect of the defects of the modulus of elasticity and of the density to the eigenvalues of bending vibrations of a sheet of paper is analyzed. The analysis is based on the relationship described in [1]. In order to obtain the graphical representation of results the procedure of conjugate approximation [2] with smoothing [3] is used for the determination of nodal values of the effects of change of the modulus of elasticity and of the density for the first eigenmodes.

In the papers [4, 5] the vibrations of paper without defects are analyzed for symmetric and un-symmetric loading of the tape of the paper. Thus the purpose of the investigation in this research paper is to determine the eigenmodes of the standing waves of the paper excited by vibrations by using the method of experimental investigations and to analyze the differences of the eigenmodes of the paper with defects and without them.

The obtained results are applicable for the automatic control of the quality of the paper.

2. Model for the analysis of defects to the vibrations of a sheet of paper

First of all the eigenproblem of bending vibrations of a sheet of paper is solved. In the theoretical study of paper sheet dynamics the following characteristics of the analyzed paper have been adopted: the r sheet was 0.2 m width and 0.2 m length, the son's ratio ν=0.3, the modulus of elasticity E=1.1 GPa, the density of the paper ρ=785 kg/m^3, the thickness h=0.0001 m.

Further x, y and z denote the axes of the system of coordinates. The sheet of paper is analyzed by using a one - dimensional model of beam type. The bending element has two nodal degrees of freedom: the displacement w in the direction of the z axis and the rotation Θ_y about the y axis. The displacement u in the direction of the x axis is expressed as $u=z\Theta_y$.

The generalized displacements are represented in the following way:

$$\begin{Bmatrix} w \\ \Theta_y \end{Bmatrix} = [N]\{\delta\}, \tag{1}$$

where $\{\delta\}$ is the analyzed eigenmode and

$$[N] = \begin{bmatrix} N_1 & 0 & \dots \\ 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the finite element.

The longitudinal strain is represented as:

$$\varepsilon_x = [B]\{\delta\}, \tag{3}$$

where:

$$[B] = \begin{bmatrix} 0 & \dfrac{dN_1}{dx} & \dots \end{bmatrix}. \tag{4}$$

The shear strain is represented as:

$$\gamma_{xz} = [\overline{B}]\{\delta\}, \tag{5}$$

where:

$$[\overline{B}] = \begin{bmatrix} \dfrac{dN_1}{dx} & N_1 & \dots \end{bmatrix}. \tag{6}$$

The effect of the change of the modulus of elasticity E to the analyzed eigenvalue λ is denoted as $d\lambda_E$ and can be determined from:

$$E d\lambda_E = \frac{E}{1-\nu^2}\frac{h^3}{12}\varepsilon_x^2 + \frac{E}{2(1+\nu)1.2}h\gamma_{xz}^2, \tag{7}$$

where ν is the Poisson's ratio and h is the thickness of the paper.

The effect of the change of the density ρ to the analyzed eigenvalue is denoted as $d\lambda_\rho$ and can be determined from

$$\rho d\lambda_\rho = -\lambda \begin{Bmatrix} w \\ \Theta_y \end{Bmatrix}^T \begin{bmatrix} \rho h & 0 \\ 0 & \rho\dfrac{h^3}{12} \end{bmatrix} \begin{Bmatrix} w \\ \Theta_y \end{Bmatrix}. \tag{8}$$

The nodal values of $E d\lambda_E$ and of $\rho d\lambda_\rho$ are denoted by $\{\delta_E\}$ and $\{\delta_\rho\}$ respectively and are determined from

$$\int\left(\left[\hat{N}\right]^T\left[\hat{N}\right]+\left[\hat{B}\right]^T\Lambda\left[\hat{B}\right]\right)dx\left[\{\delta_E\} \quad \{\delta_\rho\}\right]= \\ =\int\left[\hat{N}\right]^T\left[Ed\lambda_E \quad \rho d\lambda_\rho\right]dx, \tag{9}$$

where:

$$\left\lfloor\hat{N}\right\rfloor=\left[N_1 \quad \ldots\right], \tag{10}$$

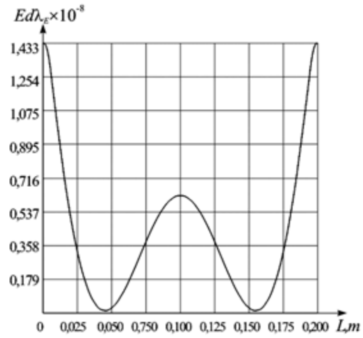
$$\left[\hat{B}\right]=\left[\frac{dN_1}{dx} \quad \ldots\right], \tag{11}$$

and Λ is the smoothing parameter.

3. Numerical results

At both ends of the paper the generalized displacements are assumed to be to zero.

The effects of defects of the modulus of elasticity and of the density to the first eigenvalue are presented in Fig.1a and Fig. 1b.



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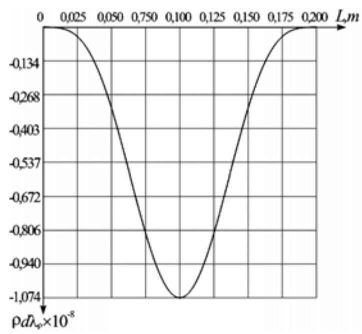
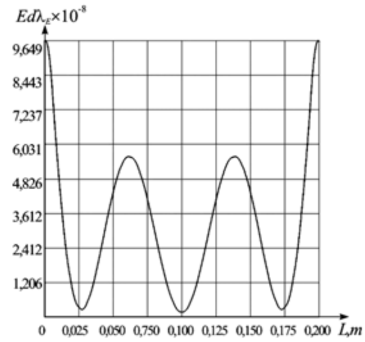


b)

Fig. 1. **Effect of the change of:a) modulus of elasticity, b) density to the first eigenvalue**

The corresponding results for the second eigenvalue are presented in Fig. 2a and in Fig. 2b, … , for the fourth eigenvalue in Fig. 4a and in Fig. 4b. On the vertical axis in all the figures a) the values of $Ed\lambda_E$ are represented and in all the figures b) the values of $\rho d\lambda_\rho$ are represented.



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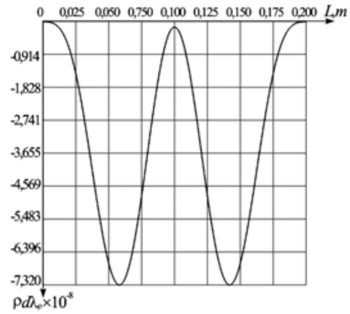

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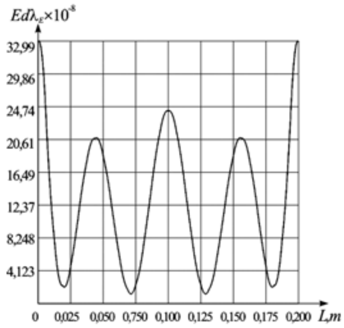
Fig. 2. **Effect of the change of: a) modulus of elasticity, b) density to the second eigenvalue**



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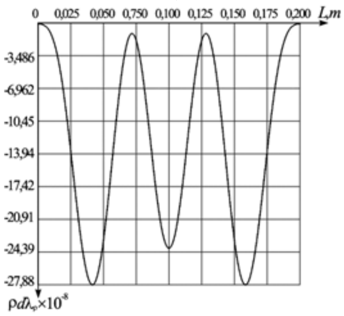
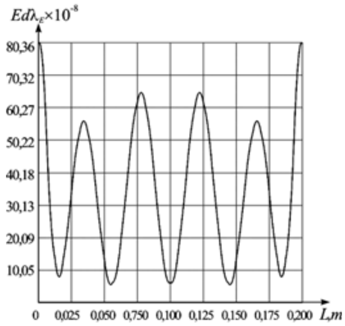
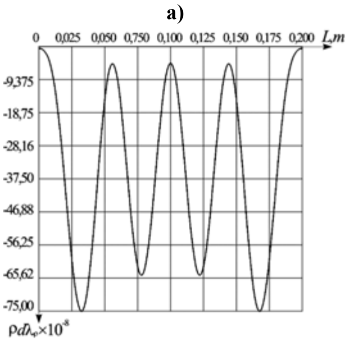


b)

Fig. 3. **Effect of the change of: a) modulus of elasticity, b) density to the third eigenvalue**



a)



b)

Fig. 4. **Effect of the change of: a) modulus of elasticity, b) density to the fourth eigenvalue.**

4. The method and the results of experimental investigation

In order to determine the dynamical characteristics of the paper a special setup for experimental investigation was used (see Fig. 5) [4, 5].

For the investigations the paper Plano Plus was chosen. Technical characteristics of this paper are: the surface density 80g/m^2, the thickness 102 μm.

The sample of paper with different density obtained during the production of the paper (see Fig. 6) was used in the investigations. The experiments were performed for the defect in the longitudinal direction of the production of the paper as well as in the transverse direction of the production of the paper by exciting the tape of paper using vibrations and loading it symmetrically with the distributed load of 25,5 N.

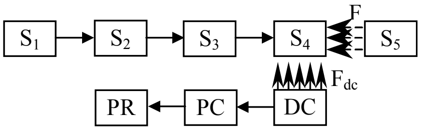


Fig.5. **Structural diagram of the setup for experimental investigations: S_1 – signal generator; S_2 – signal amplifier; S_3 – setup for the analysis of vibrations; S_4 – the investigated material; S_5 – source of monochromatic light; DC – digital camera; PC – personal computer; PR – printer; F_{dc} – light flux of the digital camera; F – flux of light projected using the source of light through the grid of the step *p* at an angle α**

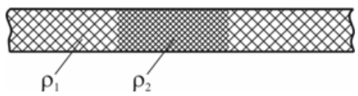


Fig. 6. **The paper Plano Plus 80 g/m^2 with a different density (ρ_1 and ρ_2 – density of the paper)**.

The obtained results of experimental investigations are presented in Fig. 7 – Fig. 10. For the paper Plano Plus for a different density in the longitudinal and transverse direction of manufacturing of the paper the image of the first eigenmode is presented in Fig. 7a and Fig. 7b. The corresponding images of the second eigenmode for different density of the paper are presented in Fig. 8a and Fig. 8b, for the third eigenmode they are presented in Fig. 9, for the fourth eigenmode they are presented in Fig. 10.

From the presented results it is seen that in the case of the paper with defects the first eigenmode (see Fig. 7) takes place at similar frequencies for the transverse as well as longitudinal directions of manufacturing, but the image of the mode itself for those cases changes (for this case in the longitudinal direction of the list of paper near to the center two elliptically shaped nodal lines occur), when compared with the first eigenmode of the paper without defects, where along the sides of the borders of the list of paper nodal lines of the shape of partial ellipse occur.

When the paper has different a density for the longitudinal direction of manufacturing of the paper this eigenmode takes place at lower frequencies when compared with the transverse direction of manufacturing of the paper (see Fig. 8a, 8b). Besides from the obtained results it is seen that the shapes of the second eigenmode of the paper with defects (see Fig. 8a and Fig. 8b) are similar to the shape of the first eigenmode of the paper without defects (see Fig. 7c and Fig. 7d).

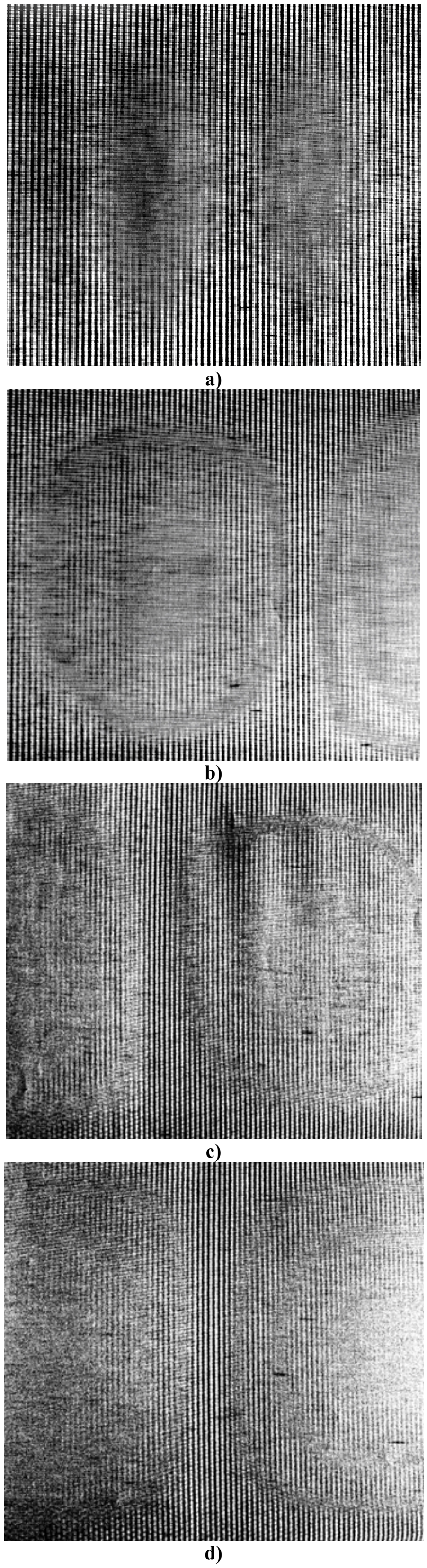
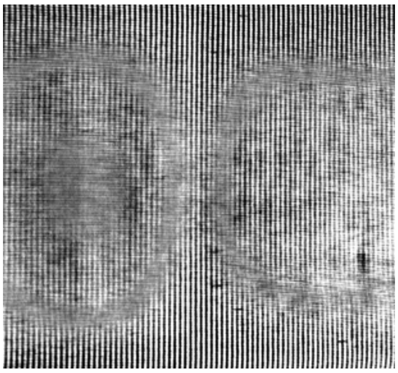


Fig. 7. **The first eigenmode of the paper Plano Plus 80 g/m^2: a) different density of the paper, longitudinal direction of production (frequency of vibrations 155 Hz, amplitude 2×10^{6}m); b) different density of the paper, transverse direction of production (frequency of vibrations 156 Hz, amplitude 2×10^{-6}m); c) paper without defects, longitudinal direction of production (frequency of vibrations 162 Hz, amplitude 2×10^{-6}m); d) paper without defects, transverse direction of production (frequency of vibrations 163 Hz, amplitude 2×10^{6}m).**



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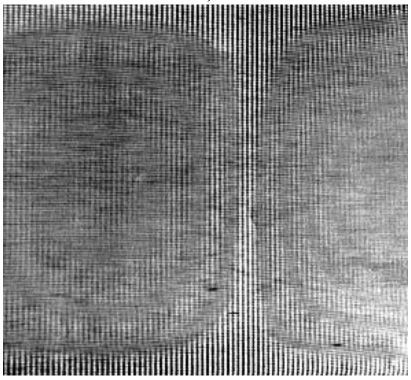

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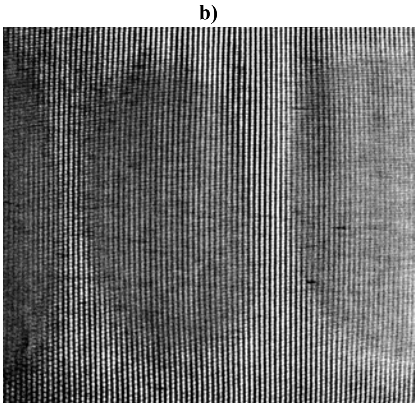

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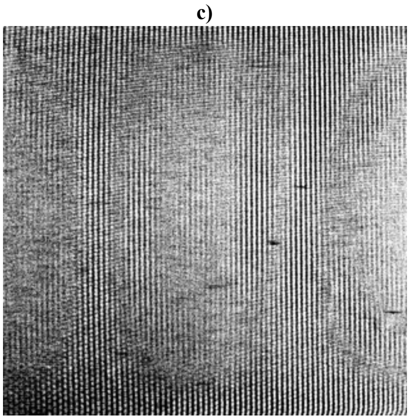
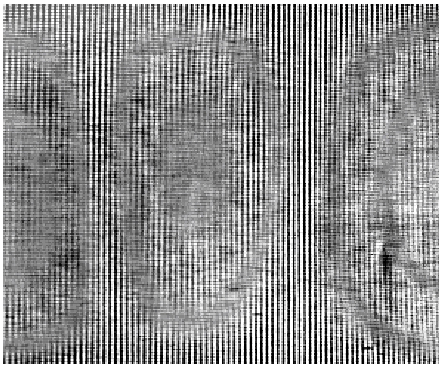


d)

Fig. 8. **The second eigenmode of the paper Plano Plus 80 g/m^2: a) different density of the paper, longitudinal direction of production (frequency of vibrations 158 Hz, amplitude 2×10^{6}m); b) different density of the paper, transverse direction of production (frequency of vibrations 174 Hz, amplitude 2×10^{6}m); c) paper without defects, longitudinal direction of production (frequency of vibrations 186 Hz, amplitude 2×10^{6}m); d) paper without defects, transverse direction of production (frequency of vibrations 180 Hz, amplitude 2×10^{6}m).**



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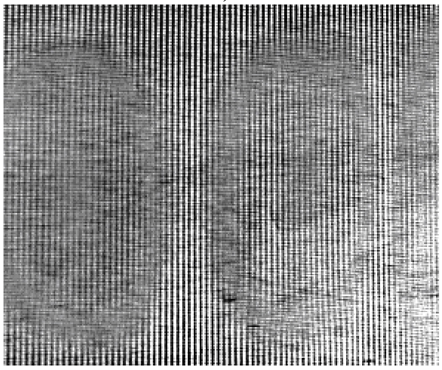

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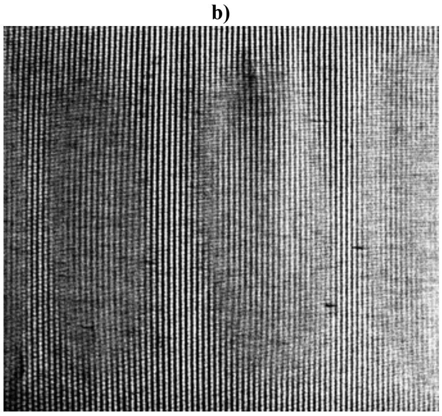

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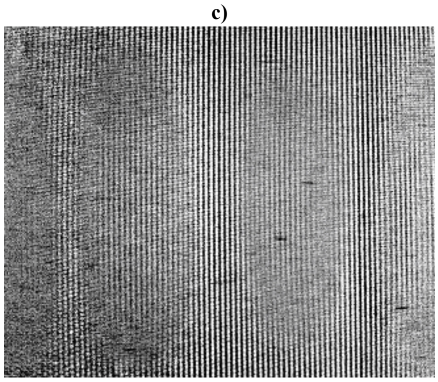
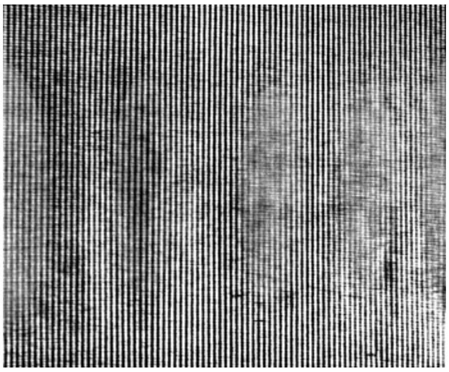
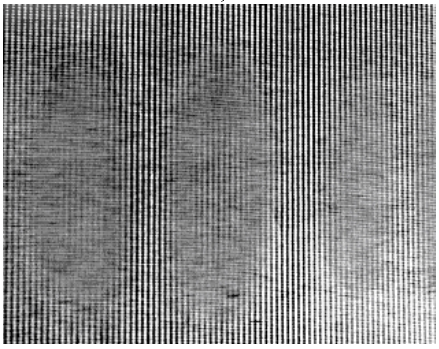


d)

Fig. 9. **The third eigenmode of the paper Plano Plus 80 g/m^2: a) different density of the paper, longitudinal direction of production (frequency of vibrations 162 Hz, amplitude 2×10^{6}m); b) different density of the paper, transverse direction of production (frequency of vibrations 176 Hz, amplitude 2×10^{-6}m); c) paper without defects, longitudinal direction of production (frequency of vibrations 194 Hz, amplitude 2×10^{-6}m); d) paper without defects, transverse direction of production (frequency of vibrations 188 Hz, amplitude 2×10^{-6}m).**



a)



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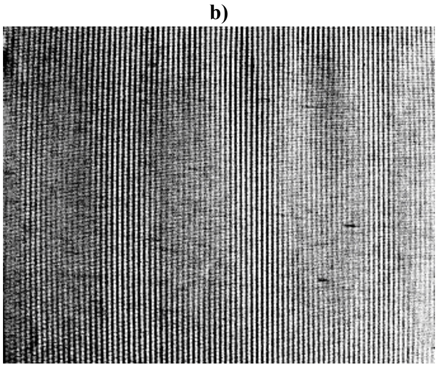

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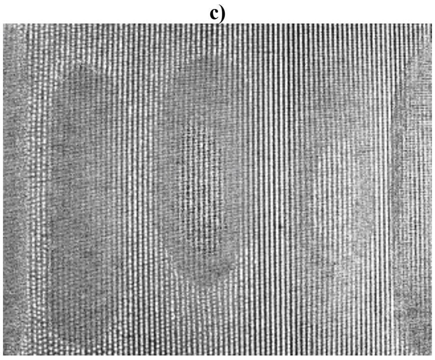

d)

Fig.10. **The fourth eigenmode of the paper Plano Plus 80 g/m^2: a) different density of the paper, longitudinal direction of production (frequency of vibrations 181 Hz, amplitude 2×10^{6}m); b) different density of the paper, transverse direction of production (frequency of vibrations 188 Hz, amplitude 2×10^{-6}m); c) paper without defects, longitudinal direction of production (frequency of vibrations 200 Hz, amplitude 2×10^{-6}m); d) paper without defects, transverse direction of production (frequency of vibrations 205 Hz, amplitude 2×10^{-6}m)**.

The shapes of the third eigenmode of the paper with defects (see Fig. 9a, Fig. 9b) are similar to the shapes of the second eigenmode of the paper without defects (see Fig. 8c and Fig. 8d). The images of the third eigenmode for the different density of the paper in the transverse as well as the longitudinal directions of production of the paper took place at lower frequencies if compared with the eigenmodes of the paper without defects.

The shapes of the fourth eigenmode of the paper with defects (see Fig. 10a and Fig. 10b) are similar to the shapes of the third eigenmode of the paper without defects (see Fig. 9c and Fig. 9d). Besides the images of the fourth eigenmode for different density of the paper in the transverse as well as the longitudinal directions of production of the paper took place at lower frequencies when compared with the eigenmodes of the paper without defects.

The proposed method of optical diagnostics of defects is applicable for automatic control of polygraphic materials (paper, cardboard and etc.).

5. Conclusions

Bending vibrations of a sheet of paper are analyzed. The effects of the defects of the modulus of elasticity and of the density to the first eigenvalues are determined.

The graphical representation of results for the first eigenmodes is obtained by using the procedure of conjugate smoothing.

On the basis of experimental investigations it is obtained that for different density of the paper the images of the first eigenmode for the longitudinal as well as transverse directions of production are obtained of different shape when compared with the first eigenmode of the paper without defects (for both the longitudinal as well as transverse directions of production). The shapes of the second eigenmode of the paper with defects are similar to the shape of the first eigenmode of the paper without defects, and also the shapes of the third eigenmode of the paper with defects are similar to the shape of the second eigenmode of the paper without defects, and the shapes of the fourth eigenmode of the paper with defects are similar to the shape of the third eigenmode of the paper without defects.

Besides the images of the first – fourth eigenmodes for different density of the paper in the transverse as well as the longitudinal directions of production of the paper took place at lower frequencies when compared with the eigenmodes of the paper without defects.

The proposed method of optical diagnostics of defects is applicable in the systems of automatic control of polygraphic materials (paper, cardboard and etc.).

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Tolygių defektų įtaka popieriaus lapo virpesiams

Reziumė

Tirta tampros modulio ir tankio defektų įtaka tikrinėms popieriaus lenkimo virpesių vertėms. Tampros modulio ir tankio pokyčių įtakos tikrinėms vertėms grafikai gauti taikant jungtinio glotninimo procedūrą.

Projekcinio muaro metodu eksperimentiškai tirtos popieriaus su defektais ir be jų virpesiais žadinamos savos formos. Pasiūlytasis optinis defektų diagnostikos metodas gali būti taikomas pakavimo medžiagų automatinės kontrolės sistemose.

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