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Investigation of dynamics of the mechatronical comparator

A. Kilikevičius[1a], V. Vekteris[2b], K. Slivinskas[3c], A. Kasparaitis[4d]

[1,2,3,4] *Department of Machines Engineering Vilnius Gediminas Technical University*

J. Basanavičiaus Str. 28 LT-2009 Vilnius, Lithuania

[a] *E-mail:* akilikevicius@gmail.com; [b] *E-mail:* vekteris@me.vgtu.lt;

[c] *E-mail:* Kastytis.Slivinskas@me.vgtu.lt; [d] *E-mail:* a.kasparaitis@bsp.lt;

Abstract

Investigation of the influence of dynamic properties of the high precision line scales calibration comparator to its accuracy is analyzed in the paper. Therefore, multi- body dynamic and mathematical models of the comparator were proposed. They allowed to evaluate characteristic sources of excitations and possibilities of rising undesirable oscillations influencing the accuracy of the comparator operation. Calculated amplitude-frequency responses and modes of oscillations at resonance frequencies allow to determine dangerous frequencies.

Key words: comparator, dynamic model, dynamic calculations, carriage, oscillations, line scale.

Introduction

When designing precise machines, which is the technical basis of high technologies, the essence property sought of the new system is its precision. A rapid progress of technologies, first of all micro- and nanotechnologies, rises higher and higher requirements to accuracy, productivity and other features of precise mechatronical systems. At the same time, it stimulates to design machines of a new quality and to improve existing machines and devices. The survey of papers [1, 2, 3, 4, 5] shows that operation principles and the type of the precise length measurement comparator carriage drives used and its design hardly influences the measurement accuracy. This fact is related with the dynamic impact on the reading quality, because when a scotch length is measured, vibrations are a significant factor influencing the reading accuracy. The precision length measurement comparator being designed [5, 6, 7, 8] is the most accurate length measurement mean in Lithuania. During analysis of dynamic errors, it is necessary to know vibration amplitude values of the carriage with the purpose to evaluate the influence of carriage drives and dampers used on the carriage vibration activity, and at the same time on the measurement accuracy.

The vibration activity of the carriage and its influence on the measurement accuracy is analyzed in the presented work.

Object of investigation

Modern line gauges are produced of various cross section shapes and length – starting from several micrometers till several meters (till 3400 mm in Lithuania [6]) by using for their production such materials as steel, glass, glass ceramics and such a nowest material as the glass-ceramics Zerodur, having an especially small expansion coefficient. Additionally the distance between lines can be various – from 0,5 µm in especially precise machines till several milimeters and more. During calibration of line gauges the distances between line centres are measured, hovever in particular cases they are measured between the edges of profiles of the lines [4]. Digital measuring microscopes enable to perform the precise positioning of the line gauge calibration systems and to estimate the quality of their lines and accuracy of the line position.

The comparator was installed into standard conditioned premises.

The precise line gauge calibration comparator under the investigation consists of nine main parts: a laser type interferometer, a gauge for measurement of environmental parameters, a microscope with CCD camera, drive system, controller, data gathering and processing computer, correction system, granite guide ways, the carriage system consisting of the force carriage and the precise carriage.

All parts are interrelated and synchronized.

The base part of the comparator is the massive fine structure granite frame of the 4 m length with guide ways, which in the horizontal plane is placed on four pneumatic supports, which dampes high frequency oscillations. The carriage system moves along the frame guide ways. The level of oscillations of the granite guide ways during measurements in the rest state did not exceed 0,7 µm at 25 Hz and 0,1 µm at other ranges of other oscillation frequencies [7]. After supplying the pressure into aerostatic bearings of carriages, the carriages slide by the help of these aerostatic bearings along six high accuracy guides. The clearance between planes of bearings and guide surfaces does not exceed 6 µm. The airflow rate is very small. The carriage system is designed in such a way, that the carriages are located on stiffly mounted aerostatic supports which are preloaded by the help of supports mounted an the opposite side, which are mounted with possibility to spring. It enables to adjust required small clearances in the aerostatic supports and to increase the rigidity.

The carriage system is pulled along the stand by the help of the friction drive controlled by a program. The carriage and the drive system construction enables eliminate the influence of the drive on the accuracy of the longitudinal motion. For this aim the carriage system is composed from the force carriage and the precise one. The drive is connected to the first part and all measuring systems, which are used to measure of carriage motions and to detect lines of the scale being calibrated, to the second one. The force carriage and the precise carriage are interconnected by the element, which is the force transmitting mean. This element is designed of such a construction, that the direction of the force transmitted by it coincides with the direction of motion of carriages on the guides (forces in the transverse direction do not act). It is important also the fact, that the element connected the force and the precise carriages is mounted in such a position, that the direction of the force transmitted by him goes through the precise carriage mass centre. This eliminates twisting moments, which can give rise the negative impact to the accuracy of the calibration.

The dynamic model and methodology of deriving of motion equations

The precision length comparator is a complicated mechatronical system, for right evaluation of which dynamic investigation should be performed. During design of such systems according to the traditional methodology based on the strength criteria [10] the protection of the system from small (of the order of micrometers or tens of nanometers) however undesirable influence of oscillation of the beam focusing point, rising as a result of the common resilient oscillations of the construction is necessary. These hardly predicted oscillations can raise due to seismic excitation of the construction, although not often the reason of their rising can be peculiarities of the machine operation, e.g., the drive system during operation can raise small oscillations, which are transmitted by the construction in dependence of its geometrical shape and dimensions, and construction particularities, which influence the rigidity properties and mass distribution. Very important is construction of guides and its rigidity, damping properties in connections having clearances, etc. Even small changes in temperature can influence the particular loosing in connections and in guides or cause thermal deformations, which finally has influence an the laser beam illuminated point position. These deformations and displacements are of a transitional nature, so they can not be eliminated by usual compensating means.

Equations of motion we will derive by dividing the comparator model into three subsystems (Fig. 1):

- the granite frame subsystem O_1 composes the granite guide ways with vibration insulation supports without relations with the carriage system;
- the force carriage system O_2 with links related it with the granite frame;
- the precise carriage system O_3 with links related it with the granite frame and the force carriage.

The connection of subsystems into the unanimous system is evaluated by the help of auxiliary links, which are expressed by the help of auxiliary coordinates.

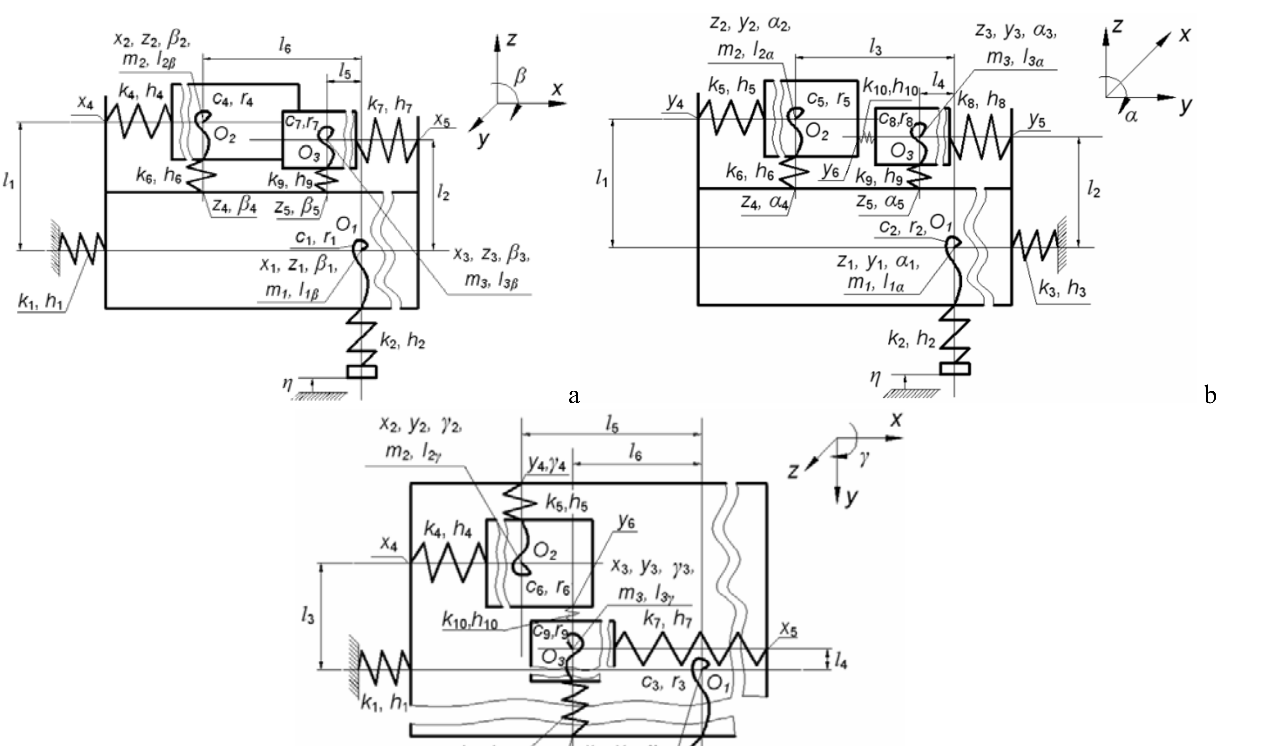

Fig. 1. **Dynamic model of the precise length measuring comparator: a - the wiev in the *zx* plane, b - the wiev in the *zy* plane, c - the wiev in the *yx* plane**

Explanations of notations used in Fig. 1: O_1, O_2, O_3 are origines of coordinate systems of separate subsystems of the model, they coincide with the mass centres of corresponding parts; m_1, m_2, m_3 are their mass centers; I_{1j}, I_{2j}, I_{3j} are their moments of inertia, where j corresponds to the twisting angles α, β, γ; k_n are the coefficients of rigidity according to linear axes of motion (n=1, 2, ... , 9); c_n are the coefficients of rigidity according to the angular motion coordinates (n=1, 2, ... , 9); h_n are damping coefficients according to the linear axes of motion (n=1, 2, ... , 9); r_n are damping coefficients for angular motion (n=1, 2, ... , 9); η expresses the kinematical excitation function (oscillations of the foundation).

The coordinates x_i, y_i, z_i, α_i, β_i, γ_i (i=1, 2, 3) we will take as the main coordinates, they fix linear and angular motions of the mass centers of subsystems in their coordinate systems. The coordinates x_4, y_4, z_4, α_4, β_4, γ_4, x_5, y_5, z_5, α_5, β_5, γ_5 are surplus (auxiliary) coordinates. There are 18 main coordinates and 12 auxiliary coordinates.

The line scales calibration comparator is a complicated dynamic system having many degrees of freedom with resilient, dissipative and others relations. To derive differential equations of it it is expedience to use the Lagrange equation of the second order:

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}_i}\right)-\frac{dT}{dq_i}+\frac{d\Phi}{d\dot{q}}+\frac{d\Pi}{dq_i}=Q_i(t) \quad (1)$$

where T, Π, Φ are the kinetic and potential energies and the disipative function of the system, $\{q\},\{\dot{q}\},\{\ddot{q}\}$ are the vectors of displacements, velocities and accelerations, and $\{Q(t)\}$ is the vector of external excitation forces.

Kinetic and potential energies of individual subsystems will be the following:

$$2T_i = m_i\left(\dot{x}_i^2+\dot{y}_i^2+\dot{z}_i^2\right)+I_{i\alpha}\dot{\alpha}_i^2+I_{i\beta}\dot{\beta}_i^2 + I_{i\gamma}\dot{\gamma}_i^2, \text{(when i = 1, 2, 3)} \quad (2)$$

$$2\Pi_1 = k_1(x_1-x_0)^2+k_3(y_1-y_0)^2+k_2(z_1-z_0)^2 + c_2(\alpha_1-\alpha_0)^2+c_1(\beta_1-\beta_0)^2+c_3(\gamma_1-\gamma_0)^2 \quad (3)$$

$$2\Pi_2 = k_4(x_2-x_4)^2+k_5(y_2-y_4)^2+k_6(z_2-z_4)^2 + c_5(\alpha_2-\alpha_4)^2+c_4(\beta_2-\beta_4)^2+c_6(\gamma_2-\gamma_4)^2 \quad (4)$$

$$2\Pi_3 = k_7(x_3-x_5)^2+k_8(y_3-y_5)^2 + k_9(z_3-z_5)^2+k_{10}(y_3-y_6)^2+c_8(\alpha_3-\alpha_5)^2 + c_7(\beta_3-\beta_5)^2+c_9(\gamma_3-\gamma_5)^2, \quad (5)$$

where $x_0, y_0, z_0, \alpha_0, \beta_0, \gamma_0$ are the amplitudes of cinematic excitations acting of the foundation according to the corresponding coordinates.

Expressions of disipative functions will have the following form:

$$2\Phi_1 = h_1(\dot{x}_1-\dot{x}_0)^2+h_3(\dot{y}_1-\dot{y}_0)^2+h_2(\dot{z}_1-\dot{z}_0)^2 + r_2(\dot{\alpha}_1-\dot{\alpha}_0)^2+r_1(\dot{\beta}_1-\dot{\beta}_0)^2+r_3(\dot{\gamma}_1-\dot{\gamma}_0)^2, \quad (6)$$

$$2\Phi_2 = h_4(\dot{x}_2-\dot{x}_4)^2+h_5(\dot{y}_2-\dot{y}_4)^2+h_6(\dot{z}_2-\dot{z}_4)^2 + r_5(\dot{\alpha}_2-\dot{\alpha}_4)^2+r_4(\dot{\beta}_2-\dot{\beta}_4)^2+r_6(\dot{\gamma}_2-\dot{\gamma}_4)^2, \quad (7)$$

$$2\Phi_3 = h_7(\dot{x}_3-\dot{x}_5)^2+h_8(\dot{y}_3-\dot{y}_5)^2 + h_9(\dot{z}_3-\dot{z}_5)^2+h_{10}(\dot{y}_3-\dot{y}_6)^2+r_8(\dot{\alpha}_3-\dot{\alpha}_5)^2 + r_7(\dot{\beta}_3-\dot{\beta}_5)^2+r_9(\dot{\gamma}_3-\dot{\gamma}_5)^2. \quad (8)$$

The mathematical model of the total system will compose the system of differential equations of the second order and algebraic constraint equations:

a) the system of differential equations

$$[A]\{\ddot{q}\}+[B]\{\dot{q}\}+[C]\{q\}=\{Q(t)\}. \quad (9)$$

b) constraint equations:

$$\begin{aligned}
&x_4-x_1-l_1\beta_1-l_3\gamma_1=0; y_4-y_1-l_1\alpha_1-l_6\gamma_1=0;\\
&z_4-z_1-l_3\alpha_1+l_6\beta_1=0; x_5-x_1-l_1\beta_1-l_4\gamma_1=0;\\
&y_5-y_1-l_1\alpha_1-l_5\gamma_1=0; z_5-z_1-l_3\alpha_1+l_5\beta_1=0;\\
&\alpha_4-\alpha_1=0; \beta_4-\beta_1=0; \gamma_4-\gamma_1=0;\\
&\alpha_5-\alpha_1=0; \beta_5-\beta_1=0; \gamma_5-\gamma_1=0.
\end{aligned} \quad (10)$$

Here $[A]$, $[B]$, $[C]$ are the matrices of inertia, damping and rigidity; $\{q\},\{\dot{q}\},\{\ddot{q}\}$ are the vectors of displacements, velocities and accelerations, $\{Q(t)\}$ is the vector of external forces, including forces due to kinematical excitation $\{Q(t)\}=\{Q_{extern.}(t)\}+\{Q_{q0}(t)\}$.

The matrices $[A]$, $[B]$, $[C]$, the vectors $\{q\},\{\dot{q}\},\{\ddot{q}\}$ and the vector of forces $\{Q(t)\}$ have following compositions:

$$[A]=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1\,30}\\ a_{21} & a_{22} & \cdots & a_{2\,30}\\ \vdots & \vdots & \vdots & \vdots\\ a_{30\,1} & a_{30\,2} & \cdots & a_{30\,30}\end{bmatrix}; \quad (11)$$

$$[B]=\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1\,30}\\ b_{21} & b_{22} & \cdots & b_{2\,30}\\ \vdots & \vdots & \vdots & \vdots\\ b_{30\,1} & b_{30\,2} & \cdots & b_{30\,30}\end{bmatrix}; \quad (12)$$

$$[C]=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1\,30}\\ c_{21} & c_{22} & \cdots & c_{2\,30}\\ \vdots & \vdots & \vdots & \vdots\\ c_{30\,1} & c_{30\,2} & \cdots & c_{30\,30}\end{bmatrix}; \quad (13)$$

$$\{q\}=\begin{Bmatrix} q_1\\ q_2\\ \vdots\\ q_{30}\end{Bmatrix};\ \{\dot{q}\}=\begin{Bmatrix} \dot{q}_1\\ \dot{q}_2\\ \vdots\\ \dot{q}_{30}\end{Bmatrix};\ \{\ddot{q}\}=\begin{Bmatrix} \ddot{q}_1\\ \ddot{q}_2\\ \vdots\\ \ddot{q}_{30}\end{Bmatrix}; \quad (14)$$

$$\{Q(t)\}=\begin{Bmatrix} Q_1(t)\\ Q_2(t)\\ \vdots\\ Q_{30}(t)\end{Bmatrix}; \quad (15)$$

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The data required for modeling – the stiffness of aerostatic supports, rigidity of aerostatic bearings of the carriages, rigidity of the drive system connecting the force carriage with the frame, and the rigidity of the element connecting the force carriage with the precise carriage, damping coefficients, amplitudes of kinematical excitation of the frame rising due to oscillations of the foundation – were determined experimentally. For this aim oscillations excited by a hammer impact at the characteristic points of both carriages were analyzed, also oscillations of the seismic nature at the frame supports were measured and analyzed. On the base of it accepted mathematical model coefficients were checked, especially damping coefficients.

The damping coefficients we evaluate as a result of the viscous friction for the entire model. Additionally, for the connection of the force and precise carriages we took the structural friction evaluation, which states that the friction force is proportional to the displacement and it is evaluated in most cases in mechanical connections. It gives complex coefficients in the stiffness matrix of the mathematical model.

The following main excitations, which are related to dynamics and may have influence on the comparator calibration accuracy are found in the comparator operation environment: the seismic oscillations of the foundation and excitation forces rising during the drive system operation. The first source was evaluated by measuring seismic oscillations of the foundation and after its spectral analysis amplitudes and frequencies were determined. It gives the kinematical excitation. The second source was evaluated as the excitation force added to the force carriage in the y_2 direction – this direction coincide with the acting force direction and has the most important direction related with the calibration accuracy.

Measurements were performed using the measurement equipment shown in Fig. 2.

Fig. 2. **Equipment used for impact excitation ("Brüel & Kjær"): 1) the impact hammer 8202 with the force transducer 8200, 2) the amplifier 2626, 3) the portable measurement results processing station „Machine Diagnostics Toolbox Type 9727“ ("Brüel & Kjær")**

Typical example of measurement data using this equipment is given in Fig. 3.

Calculation results

Calculation of amplitude frequency responses was carried out using the created dynamic model of the comparator. Main coordinates, which are mostly related with the comparator accuracy, are y coordinates. Just this direction coincides with the measuring direction during calibration. That is why we analyze oscillations along y coordinates. The calculation results are shown in Figs. 4-8. The system response to the most important excitation – the force added to the force carriage is shown in Fig. 4. There the amplitude-frequency responses of the force carriage (1), the precise carriage (2) and the frame (3) are shown. Amplitude-frequency responses of the force carriage with respect to the precise one (1) and of oscillations of the frame with respect to the precise carriage are shown in Fig. 5.

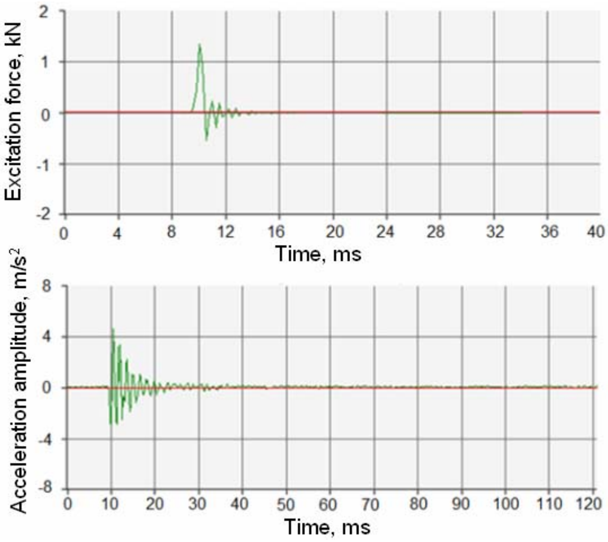

Fig. 3 **Typical data being measured**

There are seen two resonance’s zones on the frequency curves: the zone of 20-25 Hz and the zone of 280-310 Hz. The error depends on the actual excitation force amplitude. There it is taken as the unit force. Modes of oscillations falling into these zones are shown in Fig. 6. It is seen, that oscillations along the coordinates y_2 and y_3 at these frequencies are of opposite phases.

The investigation of the seismic oscillations shows that the spectrum of seismic oscillations process has main frequencies of 0,5-1,5 Hz, although additives of higher frequency can reach 18-25 Hz (Fig. 7).

The direction of the seismic excitation mainly coincides with z_1 direction and in the case of the symmetrical action of all four aerostatic supports and evenness of rigidity of them will cause oscillations of carriages also in z_2 and z_3 directions, which are not so important to the comparator action. A worse case is when excitations acting supports at the one end of the comparator differ from excitations acting supports at the second end of the comparator. These cause oscillations according to coordinate β_1 of the rotational motion, which give additives along the y axes. These frequencies can fall into the first resonance zone. We calculated the amplitude-frequency response to this excitation (Fig. 8), taken its amplitude similar to that which was measured from the spectrum, so the result obtained is close to the actual one.

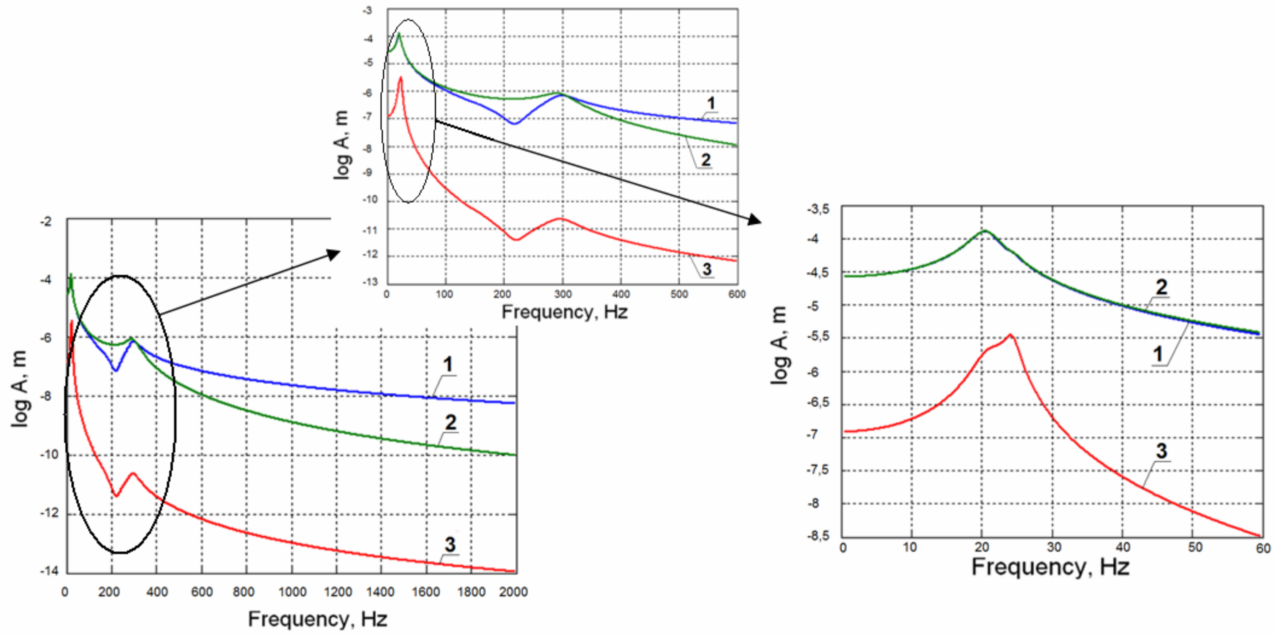

Fig. 4. Amplitude-frequency response to the excitation added to the force carriage in the y_2 direction: of the precise carriage along the coordinate y_3 (1), of the force carriage along the coordinate y_2 (2), the frame along the coordinate y_1 (3)

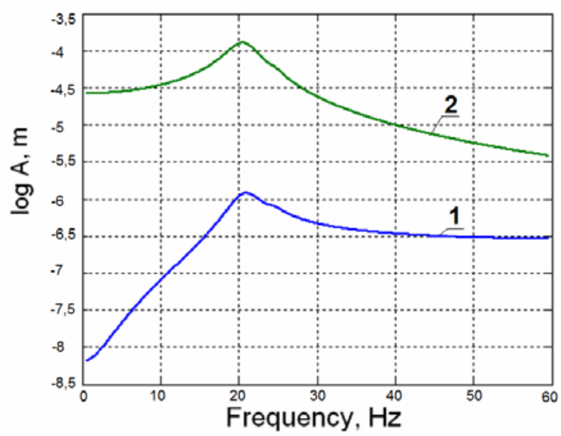

Fig. 5. **Amplitude-frequency response to the excitation added to the force carriage in the y_2 direction: of the force carriage in respect to the precise carriage, y_2-y_3 (1), of the precise carriage in respect to the frame, y_3-y_1 (2)**

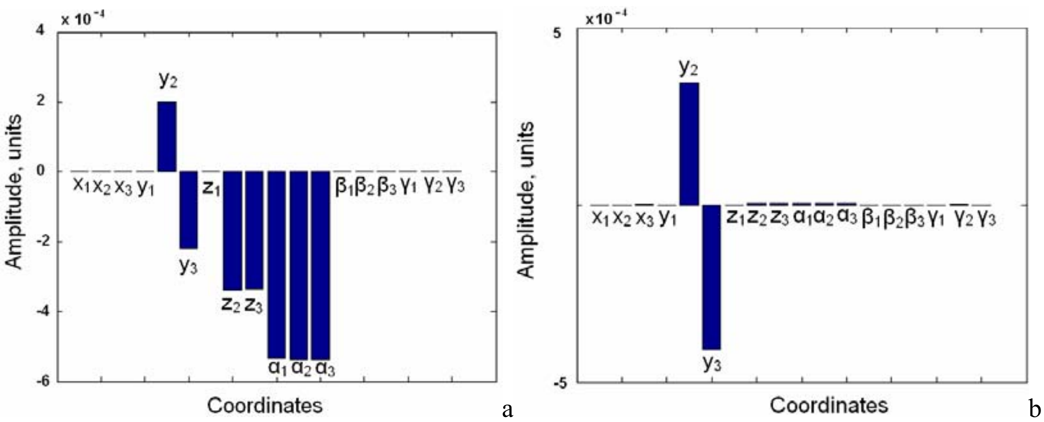

Fig. 6. **Modes of oscillations of the system at the frequency 24 Hz (a) and at the frequency 296 Hz (b)**

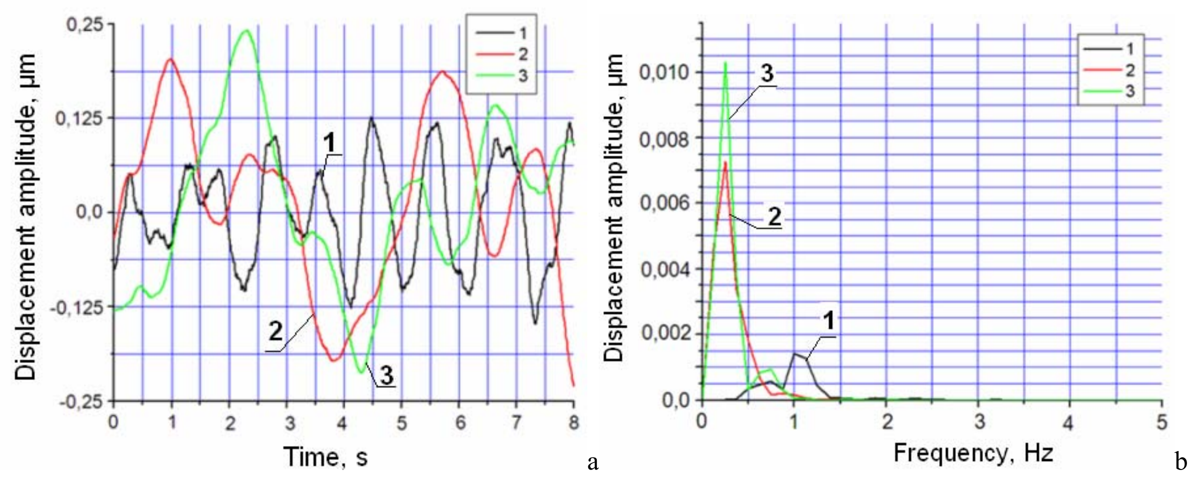

Fig. 7. **Signal of the vibration displacement (a) and its spectrum (b): 1 – vertical direction, 2 – longitudinal direction, 3 – transverse direction**

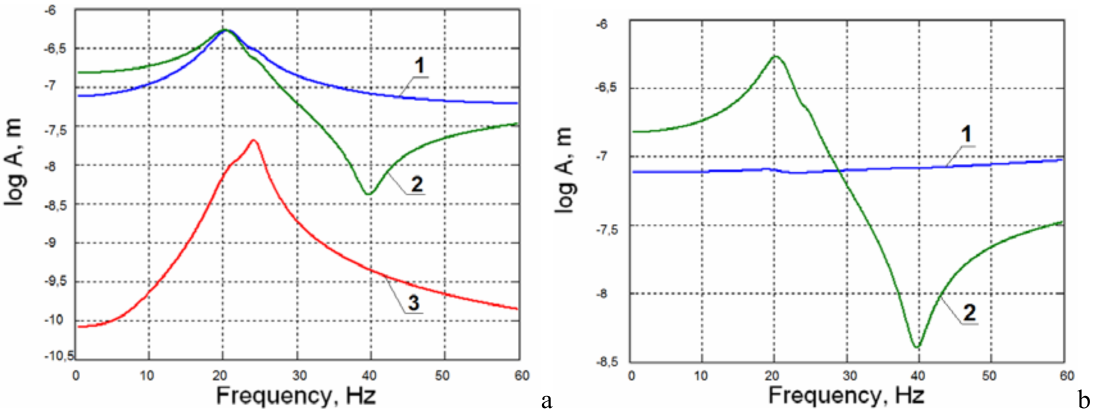

Fig. 8. **Amplitude-frequency response to the excitation forces added to the frame in the y_1 and $\alpha_1$directions: a) amplitudes of the responses of the force carriage along the coordinate y_2; (2), amplitudes of responses of the precise carriage along the coordinate y_3 (3) amplitudes of the responses of the frame along the coordinate y_1; b) amplitude of the responses of the force carriage with respect to the precise carriage, y_2-y_3 (1), and the precise carriage with respect to the frame, y_3-y_1 (2)**

As it is seen from Fig. 8, amplitude of the most important oscillations – the force carriage with respect to the precise carriage, y_2-y_3 has an enough low level which does not changes significantly in the whole range of the low frequencies. Although oscillations according coordinate y_3 with respect to the frame y_3-y_1 has the resonance with a significantly increased amplitude (Fig.8, curve 2).

Conclusions

1. The created dynamical model of the comparator system allows to analyze its characteristic dynamic oscillations and their influence on the comparator accurcy.

2. It was found, that oscillations created by the seismic kinematic excitation with the exsisting foundation movement amplitudes are small and do not have significant influence on the comparator accuracy.

3. Oscillations caused by the drive system can significantly influence the comparator operation accuracy and the corresponding attention should be paid during selection of the drive system.

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A. Kilikevičius, V. Vekteris, K. Slivinskas, A. Kasparaitis

Dinaminiai mechatroninio komparatoriaus tyrimai

Reziumė

Tirta precizinio ilgio matavimo komparatoriaus dinaminių savybių įtaka jo tikslumui. Tam buvo sukurti komparatoriaus daugelio laisvės laipsnių dinaminis ir matematinis modeliai, leidžiantys įvertinti būdingojo žadinimo šaltinius ir nepageidaujamų virpesių, darančių įtaką komparatoriaus veikimo tikslumui, kilimo galimybes. Apskaičiuotos dažninės amplitudės charakteristikos, priklausančios nuo realių, komparatoriaus darbo metu vykstančių žadinimų, taip pat virpesių formos, kas leido nustatyti komparatoriaus darbui pavojingus dažnius. Darbo rezultatai išnagrinėti ir pateiktos išvados.

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