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Non-destructive analysis of defects and vibrations of a sheet of paper

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Abstract

The effect of the defects of the modulus of elasticity and of the density of the material to the eigenvalues of bending vibrations of a sheet of paper is analyzed. Representations of the effects of change of the modulus of elasticity and of the density of the material by intensity for the first eigenmodes are determined by using the procedure of conjugate smoothing.

A simple algorithm for processing of stroboscopic geometric moiré images is proposed. It enables to visualize the moiré fringes more clearly and thus to perform the interpretation of experimental results with higher precision.

Keywords: non-destructive testing, paper, bending vibrations, defect, eigenmode, eigenvalue, conjugate approximation, finite elements, conjugate smoothing, stroboscopic moiré, geometric moiré, image processing, intensity mapping, experimental results.

Introduction

Nondestructive testing (NDT) provides valuable information to evaluate up-to-date structural properties. The principle of vibration-based NDT is to estimate the change of structural properties by investigating the change of structural vibration characteristics. The commonly used structural vibration characteristics include eigenfrequencies, mode shapes, mode shape curvatures, flexibility matrix, modal strain energy, etc. Also this makes it possible to evaluate the mechanical evolution of material parameters with time dependant properties, particularly throughout fatigue tests [1, 2].

Some investigators have studied the detection and location of defects in multi – layer materials [3, 4]. The procedure of diagnostics of uniformity of loading of a sheet of paper by using time averaged projection moiré was presented in [5]. One – dimensional continuous defects of a sheet of paper were analyzed in [6].

In this work the effect of the defects of the modulus of elasticity and of the density of the material to the eigenvalues of bending vibrations of a sheet of paper is analyzed. This differs from the investigation presented in the previously mentioned paper in the fact that the aim of this work is to analyze the two – dimensional continuous defects of a sheet of paper.

The analysis is based on the relationship described in [7]. In order to obtain the graphical representation of results the procedure of conjugate approximation [8] with smoothing [9] is used for determination of nodal values of the effects of change of the modulus of elasticity and of the density of the material for the first eigenmodes.

A simple algorithm for processing of stroboscopic geometric moiré images on the basis of the material described in [10, 11] is proposed. It enables to visualize the moiré fringes more clearly and thus to perform the interpretation of experimental results with a higher precision.

The obtained results are used in the process of design of elements of packages.

Model for the analysis of the effect of defects to the vibrations of a sheet of paper

First of all the eigenproblem of bending vibrations of a sheet of paper is solved.

Further x, y and z denote the axes of the system of coordinates. The plate bending element has three nodal degrees of freedom: the displacement w in the direction of the z axis, the rotation Θ_x about the x axis and the rotation Θ_y about the y axis. The displacements u and v in the directions of the axes x and y are expressed as $u=z\Theta_y$ and $v=-z\Theta_x$.

The generalized displacements are represented in the following way:

$$\begin{Bmatrix} w \\ \Theta_x \\ \Theta_y \end{Bmatrix} = [N]\{\delta\}, \tag{1}$$

where $\{\delta\}$ is the analyzed eigenmode and

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the finite element.

The strains in the plane of the sheet of paper are represented as:

$$\{\varepsilon\} = [B]\{\delta\}, \tag{3}$$

where:

$$[B]=\begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \ldots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \ldots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}. \tag{4}$$

The corresponding stresses are:

$$\{\sigma\}=[D]\{\varepsilon\}, \tag{5}$$

where

$$[D]=\frac{h^3}{12}\begin{bmatrix} \frac{E}{1-v^2} & \frac{Ev}{1-v^2} & 0 \\ \frac{Ev}{1-v^2} & \frac{E}{1-v^2} & 0 \\ 0 & 0 & \frac{E}{2(1+v)} \end{bmatrix}, \tag{6}$$

where h is the thickness of the paper, E is the modulus of elasticity and v is the Poisson's ratio.

The bending shear strains are represented as:

$$\{\bar{\varepsilon}\}=[\bar{B}]\{\delta\}, \tag{7}$$

where

$$[\bar{B}]=\begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & \ldots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \ldots \end{bmatrix}. \tag{8}$$

The corresponding stresses are:

$$\{\bar{\sigma}\}=[\bar{D}]\{\bar{\varepsilon}\}, \tag{9}$$

where

$$[\bar{D}]=\frac{Eh}{2(1+v)1.2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{10}$$

The effect of the change of the modulus of elasticity E to the analyzed eigenvalue λ is denoted as $d\lambda_E$ and can be determined from

$$Ed\lambda_E=\{\varepsilon\}^T\{\sigma\}+\{\bar{\varepsilon}\}^T\{\bar{\sigma}\}. \tag{11}$$

The effect of the change of the density of the material ρ to the analyzed eigenvalue is denoted as $d\lambda_\rho$ and can be determined from:

$$\rho d\lambda_\rho=-\lambda\begin{Bmatrix} w \\ \Theta_x \\ \Theta_y \end{Bmatrix}^T\begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix}\begin{Bmatrix} w \\ \Theta_x \\ \Theta_y \end{Bmatrix}. \tag{12}$$

The nodal values of $Ed\lambda_E$ and of $\rho d\lambda_\rho$ are denoted by $\{\delta_E\}$ and $\{\delta_\rho\}$ respectively and are determined from:

$$\int\left(\lfloor\hat{N}\rfloor^T\lfloor\hat{N}\rfloor+\lfloor\hat{B}\rfloor^T\Lambda\lfloor\hat{B}\rfloor\right)dxdy\left[\{\delta_E\}\ \{\delta_\rho\}\right]=$$

$$=\int\lfloor\hat{N}\rfloor^T\left[Ed\lambda_E\ \ \rho d\lambda_\rho\right]dxdy, \tag{13}$$

where

$$\lfloor\hat{N}\rfloor=[N_1\ \ldots.], \tag{14}$$

$$\lfloor\hat{B}\rfloor=\begin{bmatrix} \frac{\partial N_1}{\partial x} & \ldots \\ \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}, \tag{15}$$

and Λ is the smoothing parameter.

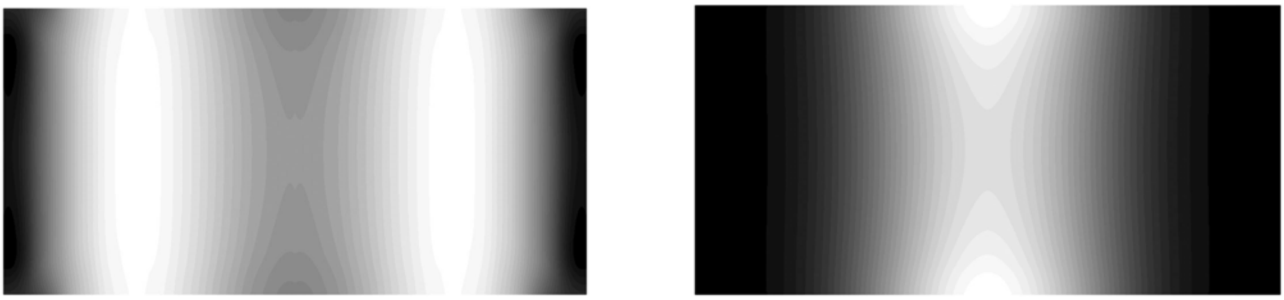
Results of analysis of two – dimensional continuous defects

At the left and the right boundaries of the rectangular sheet of paper all the generalized displacements are assumed equal to zero.

The effects of defects of the modulus of elasticity and of the density of the material to the first eigenvalue are presented in Fig. 1a and Fig. 1b. The corresponding results for the second eigenvalue are presented in Fig. 2a and in Fig. 2b, … , for the fourth eigenvalue in Fig. 4a and in Fig. 4b. In all the figures:

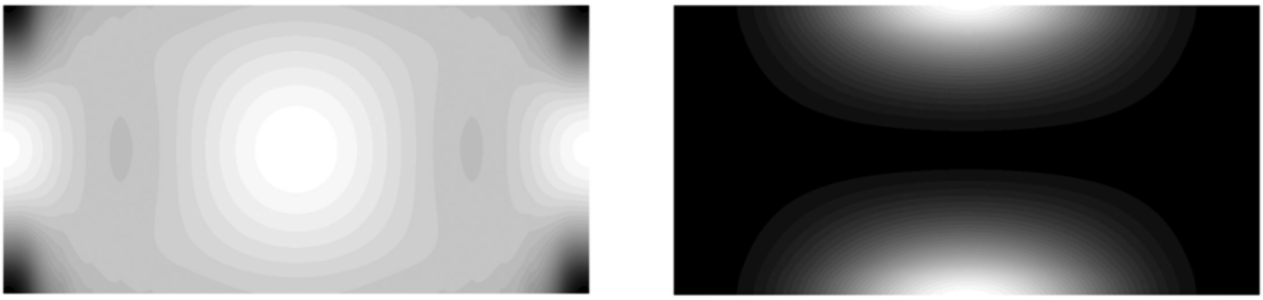
a - the values of $Ed\lambda_E$ are represented by intensity;

b - the values of $\rho d\lambda_\rho$ are represented by intensity.



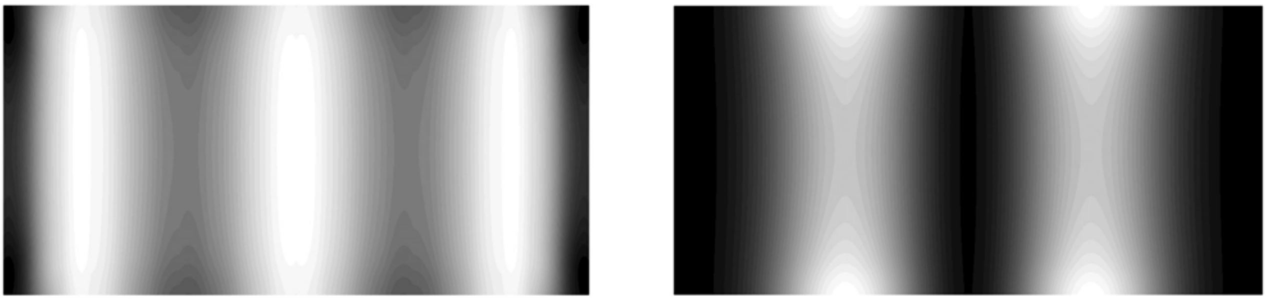
a b

Fig. 1. **Effect of the change of: a - modulus of elasticity; b - density to the first eigenvalue**.



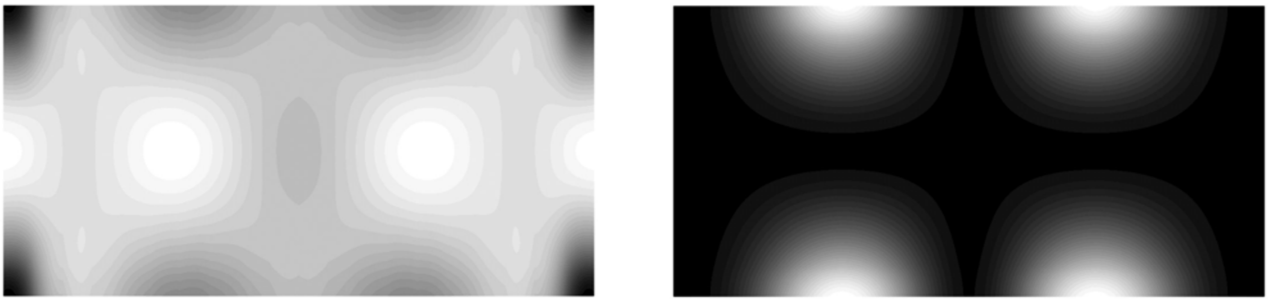
a b

Fig. 2. **Effect of change of: a - modulus of elasticity; b - density to the second eigenvalue**.



a b

Fig. 3. **Effect of change of: a - modulus of elasticity; b - density to the third eigenvalue**.



a b

Fig. 4. **Effect of change of: a - modulus of elasticity; b - density to the fourth eigenvalue**.

Procedure of processing of moiré images of vibrating elastic structures

First the following intensity mapping is performed:

$$\bar{I}(i,j) = \begin{cases} \dfrac{I(i,j)-\left(\dfrac{1}{2}-\Delta\right)}{\Delta}, & \text{when } \dfrac{1}{2}-\Delta < I(i,j) \le \dfrac{1}{2}, \\ \dfrac{\dfrac{1}{2}+\Delta-I(i,j)}{\Delta}, & \text{when } \dfrac{1}{2} \le I(i,j) < \dfrac{1}{2}+\Delta, \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

where i=0, 1, …, n_x denotes the column of pixels in the digital image and j=0, 1, …, n_y denotes the row of pixels in the digital image, $I(i, j)$ denotes the intensity of the pixel (i, j), Δ is the parameter of the intensity mapping. The intensity is assumed to be between zero and unity, zero intensity corresponds to white color, while unit intensity corresponds to black color.

Then the following procedure is performed:

$$\bar{\bar{I}}(i,j) = \begin{cases} 0, & \text{when } \quad \bar{I}_{N4}(i,j)=0 \quad \text{or} \\ & \qquad\quad \bar{I}_{ND}(i,j)=0, \\ \bar{I}(i,j), & \text{otherwise}, \end{cases} \tag{17}$$

where by $\bar{I}_{N4}(i,j)=0$ it is understood that:

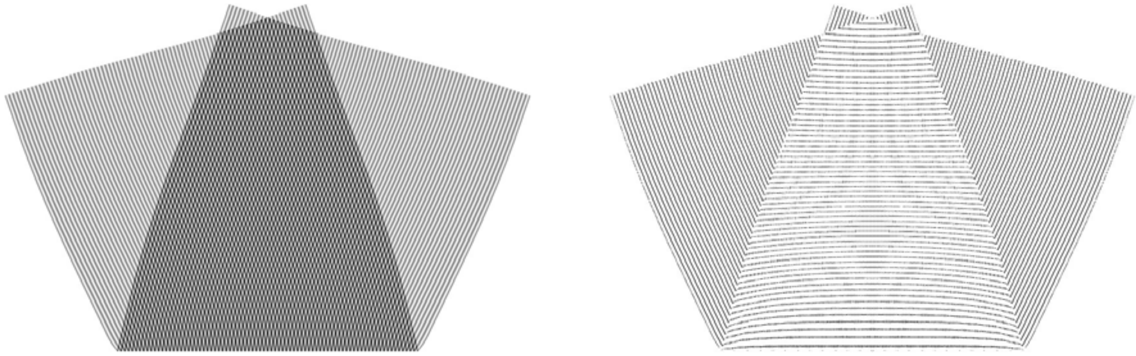
$$\bar{I}(i+1,j)=\bar{I}(i,j-1)=\bar{I}(i-1,j)=\bar{I}(i,j+1)=0, \tag{18}$$

while by $\bar{I}_{ND}(i,j)=0$ it is understood that:

$$\bar{I}(i+1,j-1)=\bar{I}(i-1,j-1)=\bar{I}(i-1,j+1)= \\ =\bar{I}(i+1,j+1)=0. \tag{19}$$

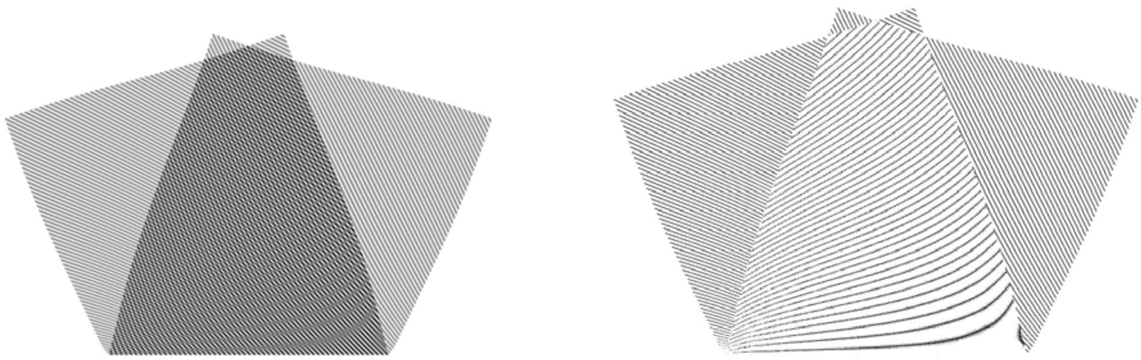
Results of processing of moiré images of vibrating elastic structures

Stroboscopic geometric moiré images for three different directions of fringes are shown in Fig. 5a, 6a and 7a. The corresponding processed images are shown in Fig. 5b, 6b and 7b.



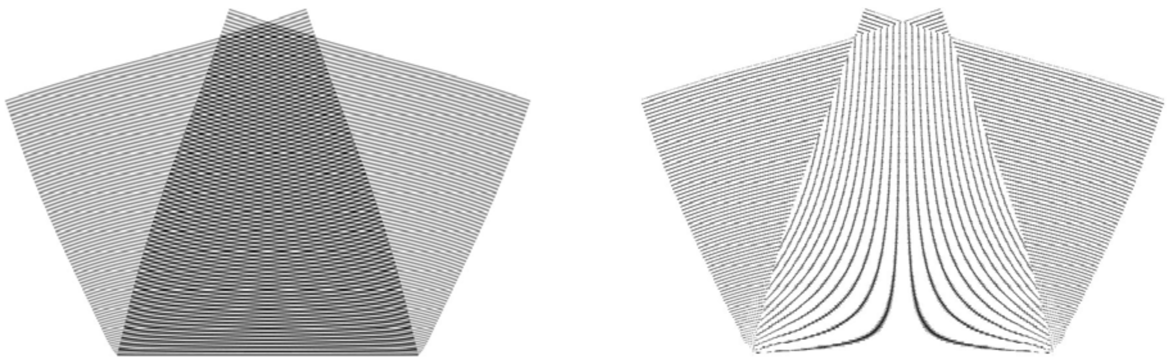
a b

Fig. 5. **Stroboscopic geometric moiré image for the first direction of fringes: a - the image, b - the processed image**



a b

Fig. 6. **Stroboscopic geometric moiré image for the second direction of fringes: a - the image, b - the processed image**



a b

Fig. 7. **Stroboscopic geometric moiré image for the third direction of fringes: a - the image, b - the processed image**.

Conclusions

Bending vibrations of a sheet of paper are analyzed. The effects of the defects of the modulus of elasticity and of the density of the material to the first eigenvalues are determined.

The representation of results by intensity for the first eigenmodes is obtained by using the procedure of conjugate smoothing.

A simple algorithm for processing of stroboscopic geometric moiré images is proposed. First the intensity mapping which depends on the chosen parameter of the procedure is performed. Then the recalculation of intensities on the basis of the values of the intensities of the neighboring pixels is performed.

The presented results show that the proposed procedure enables to visualize the moiré fringes more clearly and thus to perform the interpretation of experimental results with a higher precision.

The obtained results are used in the process of design of elements of packages.

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Neardomoji popieriaus lapo defektų ir virpesių analizė

Reziumė

Nagrinėta tampros modulio ir medžiagos tankio defektų įtaka tikrinėms popieriaus lenkimo virpesių vertėms. Tampros modulio ir tankio pokyčių įtakos tikrinėms vertėms vaizdai gauti taikant jungtinio glotninimo procedūrą. Pasiūlytas algoritmas stroboskopiniams geometrinio muaro vaizdams apdoroti. Šis metodas leidžia daug ryškiau vizualizuoti muaro juostas ir padidina eksperimentinių rezultatų apdorojimo tikslumą. Tyrimų rezultatai taikomi pakuočių elementams projektuoti.

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