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An impact of acoustical waves on a cylindrical shell

D. Gužas

Šiauliai University

E-mail: danielius.guzas@fondai

Abstract

The stressed state that emerged on the medial surface of a cylindrical shell, located in the acoustical medium and subjected to the action of a plane harmonic wave, is studied. The influence of the medium on the flexural stress on the medial surface of the shell is analyzed. The solution of the plane problem is reduced to computational formulae and charts. Moreover, computations are performed with respect to both low- and high-frequency vibrations.

Keywords: Acoustical waves, cylindrical shell.

Introduction

In the present-day developing technological processes, architectural structures, which acquire a shape of a cylindrical shell, are affected from outside by acoustical waves. External audible and inaudible sound waves, which meet on their way the cylindrical shell surfaces, excite the reflected waves that are radiated to the environment. In our works [1, 2, 3] it was studied how the sound waves affect the cylindrical shells from inside.

Here, an analysis was made how the walls of the cylindrical shell are affected by the propagation of a wave in the pipe in the diffusion and plane wave fields. However, in the present-day development of technology, it is necessary to know the dynamic stability of cylindrical shells when acoustic waves are acting outside of them and how their fluctuations interact in the given acoustic medium.

In some works that problem is being solved [1, 3]. An insignificant number of publications were dedicated to the issues of interaction of cylindrical shells with the medium. In the work [1] an axiosymmetrical problem and in [3] the general (not-axiosymmterical) problem for the cylindrical shell [2-7] are investigated

In this work, an analysis is given of the stressed state of the cylindrical shells.

Interaction of acoustical waves with a cylindrical shell

Statement of the problem. The plane harmonic longitudinal wave with a circular frequency ω is propagating in an infinite acoustical medium and meeting a cylindrical shell. The incident wave produces the reflected and radiated waves. The potential of the plane longitudinal wave has the form

$$\Phi^0 = Ae^{i(\alpha\chi - \omega t)},\ \mathrm{A} = const\ . \qquad (1)$$

where $\alpha = \omega / c_1^I$ is the wave number, ω is the circular frequency, c_1^I is the speed of the propagation of longitudinal waves in the acoustical medium.

The Eq. 1 may be presented in the polar coordinates r, θ by means of a series [2]

$$\Phi^0 = A\sum_{n=0}^{\infty} l_n i^n J_n(\alpha r)\cos n\theta e^{-i\omega t}, \mathrm{l}_0 = 1, \mathrm{l_n} = 2, \mathrm{n} \geq 1\ . \qquad (2)$$

where $J_n(\acute{\alpha},r)$ is the Bessel's function. The reflected and radiated waves, the potential of which satisfies the condition of radiation at $r \to \infty$, have the form of

$$\Phi^* = \sum_{n=0}^{\infty} A_n H_n^{(1)}(\alpha r)\cos n\theta e^{-i\omega t}\ . \qquad (3)$$

Here, A_n are the unknown coefficients, $H_n^{(1)}(\alpha r)$ is the Hankel's function of the first kind. The total field in the shell is determined by the potential

$$\Phi = \Phi^0 + \Phi^*, \qquad (4)$$

the hydrodynamic pressure, acting on the shell

$$P_b = -\rho\frac{\partial\Phi}{\partial t}, \qquad (5)$$

where ρ is the density of acoustical medium.

Let's consider the circular isotropic cylindrical shell of the constant thickness h, with the radius R, along the axis of which we will direct the coordinate axis $0Z_3$. For a plane problem with the application of the Kirchhoff-Love hypothesis, the main equations of movement of the shell in the polar coordinates r, θ in the displacements we may represent in the following form

$$\left(\frac{h^2}{12R^4}\frac{\partial^4}{\partial\theta^4} + \frac{1}{R^2}\right)U_r + \left(\frac{1}{R^2}\frac{\partial}{\partial\theta}\right)U_\theta - \frac{1-\nu^2}{Eh}\left(q_r - \rho h\ddot{\mathrm{U}}_{\mathrm{r}}\right) = 0,$$

$$\left(\frac{1}{R^2}\frac{\partial}{\partial\theta}\right)U_r + \left(\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2}\right)U_\theta - \frac{1-\nu^2}{Eh}\left(q_r - \rho h\ddot{\mathrm{U}}_{\mathrm{r}}\right) = 0. \qquad (6)$$

In the case of low vibrations of the resting viscous compressed medium, we shall write the linearized equations of the Navier-Stokes movement [8]

$$\frac{\partial\vec{V}}{\partial t} + \nu'\Delta\vec{V} + \frac{1}{\rho_0'}\vec{\nabla}\rho - \frac{1}{3}\nu'\vec{\nabla}\left(\vec{\nabla}\cdot\vec{V}\right) = 0, \qquad (7)$$

a continuity equation

$$\frac{1}{\rho_0'}\frac{\delta\rho'}{\partial t} + \vec{\nabla}\cdot\vec{V} = 0\ , \qquad (8)$$

expressions for determining the components of the tensor of stresses in rectangular coordinates Z_j

$$P_{ij} = -p\delta_{ij} + \lambda'\vec{\nabla}\cdot\vec{V} + \mu'\left(\frac{\partial \upsilon_i}{\partial z_j} + \frac{\partial \upsilon_j}{\partial z_i}\right), \qquad (9)$$

an equation of state

$$\frac{\partial P}{\partial \rho'} = a_0^2\,,\ a_0 = const\,. \qquad (10)$$

Following [3], we obtain the representation for the vector $\vec{V}$ and scalars p and ρ' in the form of

$$\vec{V} = \frac{\partial}{\partial t}\left[\vec{\nabla}\psi + \vec{\nabla}\chi\overrightarrow{C_3}x_1 + \vec{\nabla}x\left(\vec{\nabla}x\overrightarrow{C_3}\chi_2\right)\right]. \qquad (11)$$

$$P = \rho_0'\left(\frac{4}{3}\nu'\Delta - \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\psi\,. \qquad (12)$$

$$\frac{\partial \rho'}{\partial t} = \frac{\rho_0'}{a_0^2}\left(\frac{4}{3}\nu'\Delta - \frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial t^2}\psi\,. \qquad (13)$$

In the case of a plane problem for the potentials we get the following equations;

$$\left[\left(1 + \frac{4}{3}\frac{\nu'}{a_0^2}\frac{\partial}{\partial t}\right)\Delta - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\psi = 0\,, \qquad (14)$$

$$\left(\frac{\partial}{\partial t} - \nu'\Delta\right)\chi_1 = 0\,. \qquad (15)$$

From Eq. 9 we obtain expressions for determining the components of the tensor of stresses in the fluid (in the circular cylindrical system of coordinates) at r = const in the following form

$$P_{rr} = -P + \lambda'\left(\frac{\partial \upsilon_2}{\partial r} + \frac{1}{r}\frac{\partial \upsilon_\theta}{\partial \theta} + \frac{\upsilon_r}{r}\right) + 2\mu'\frac{\partial \upsilon_r}{\partial r}\,,$$

$$P_{r\theta} = \mu'\left(\frac{1}{r}\frac{\partial \upsilon_r}{\partial \theta} + \frac{\partial \upsilon_\theta}{\partial r} - \frac{\upsilon_\theta}{r}\right). \qquad (16)$$

From Eq. 11 we obtain the representation of the components of the speed vector through the potentials in the following form:

$$\upsilon_r = \frac{\partial}{\partial r}\dot{\psi} + \frac{1}{r}\frac{\partial}{\partial \theta}\dot{\chi}_1\,,\ \ \upsilon_\theta = \frac{1}{r}\frac{\partial}{\partial \theta}\dot{\psi} - \frac{\partial}{\partial r}\dot{\chi}_1\,. \qquad (17)$$

Due to thin-walls of the shell, we shall satisfy the conditions of the vibrating wall on the medial surface of the shell

$$\frac{\partial U_r}{\partial t} = \frac{\partial \Phi}{\partial r}\,, \qquad (18)$$

$$\dot{U}_r = v_r\,,\ \ \dot{U}_\theta = \upsilon_\theta\,, \qquad (19)$$

$$q_r = -P_{rr} + P_b\,,\ \ q_\theta = -P_{r\theta}\,. \qquad (20)$$

In Eq. 6–20 the following notations are introduced: $U_r, U_\theta, q_r, q_\theta$ are the components of the vector of displacements and the vector of surface forces accordingly, ρ is the density of the shell material; $\upsilon_r, \upsilon_\theta, P_{rr}, P_{r\theta}$ are the components of the speed vector $\vec{V}$ and the tensor of stresses P_{ij}, ρ_0' and a_0 – is the density and speed of the sound in the fluid at the dormant state; ρ' and P_1 – are the disturbance of density and pressure in the fluid; ν' and $\mu' = \rho_0'\nu'$ are the kinematic coefficient of viscosity and the coefficient of viscosity; λ' is the second coefficient of viscosity, in respect of which the relationship $\lambda' = -\frac{2}{3}\mu'$ takes place [9].

Representation of the solution for a cylindrical shell

We shall represent the displacement of the points of the medial surface in the form [4]:

$$U_r = \sum_{n=0}^{\infty} a_n \cos n\theta e^{-i\omega t}\,,$$

$$U_\theta = \sum_{n=0}^{\infty} b_n \sin n\theta e^{-i\omega t}\,. \qquad (21)$$

Here, a_n b_n are the unknown coefficients. The solution of Eq. 14 and 15, satisfying the conditions of limitedness at r = 0, will have the form:

$$\Psi = \sum_{n=0}^{\infty} B_n J_n(\eta_1 r)\cos n\theta e^{-i\omega t}\,,$$

$$\chi_1 = \sum_{n=0}^{\infty} C_n J_n(\eta_2 r)\sin n\theta e^{-i\omega t}\,, \qquad (22)$$

where B_n, C_n are the unknown coefficients,

$$\eta_1 = \sqrt{\frac{\omega^2}{a_0^2}\left(1 - \frac{4}{3}\frac{\nu'}{a_0^2}i\omega\right)^{-1}}\,,\ \eta_2 = \sqrt{i\frac{\omega}{\nu'}}.$$

In accordance with Eq. 17, for the compounds of speed we shall obtain

$$\upsilon_r = i\omega\sum_{n=0}^{\infty}\left[B_n\eta_n' J_n'(\eta_1 r) + C_n\frac{n}{r}J_n(\eta_2 r)\right]\cos n\theta e^{-i\omega t}\,,$$

$$\upsilon_\theta = i\omega\sum_{n=0}^{\infty}\left[B_n\frac{n}{r}J_n(\eta_1 r) + C_n\eta_2 J_n'(\eta_2 r)\right]\sin n\theta e^{-i\omega t}$$

$$(23)$$

From Eq. 16 we shall have

$$P_{rr} = \sum_{n=0}^{\infty}\left\{B_n\left[\frac{2\mu' i\omega}{r^2}\left(\eta_1 r J_n'(\eta_1 r) - n^2 J_n(\eta_1 r)\right) - \rho_0'\omega^2 J_n(\eta_1 r)\right] + C_n\frac{2\mu' i\omega n}{r^2}\left[-\eta_2 r J_n'(\eta_2 r) + J_n(\eta_2 r)\right]\right\}\cos n\theta e^{-i\omega t}$$

$$P_{r\theta} = \sum_{n=0}^{\infty}\left\{B_n\frac{2\mu' i\omega n}{r^2}\left[\left(\eta_1 r J_n'(\eta_1 r) - J_n(\eta_1 r)\right)\right] + C_n\frac{\mu' i\omega}{r^2}\left[\eta_2^2 r^2 J_n''(\eta_2 r) - \eta_2 r J_n'(\eta_2 r) + n^2 J_n(\eta_2 r)\right]\right\}\sin n\theta e^{-i\omega t}$$

$$(24)$$

$$P=\sum_{n=0}^{\infty}B_n\left\{-\frac{4}{3}\mu' i\omega\left[\eta_1^2 J_n''(\eta_1 r)+\frac{\eta_1}{r}J_n(\eta_1 r)\right]+\left(\frac{4}{3}\mu' i\omega\frac{n^2}{r^2}+\rho_0'\omega^2\right)J_n(\eta_1 r)\right\}\cos n\theta e^{-i\omega t}.$$

We shall note that here the derivatives are taken by argument. The pressure on the front of a plane wave has the form:

$$P_b=\rho_0 i\omega\sum_{n=0}^{\infty}\left[Al_n i^n J_n(\alpha r)+A_n H_n^{(1)}(\alpha r)\right]\cos n\theta e^{-i\omega t}. \quad (25)$$

Determination of the intensive state in the shell

We shall determine the efforts and moment through displacements U_r, U_θ in the form of

$$N_r=\frac{Eh}{1-\nu^2}\frac{1}{R}\left[\frac{\partial U_\theta}{\partial\theta}+U_r+\frac{h^2}{12R^2}\left(\frac{\partial^2 U_r}{\partial\theta^2}+U_r\right)\right],$$

$$M_r=\frac{D}{R^2}\left(U_r+\frac{\partial^2 U_r}{\partial\theta^2}\right), \quad (26)$$

where E is the Young's modulus, D is the cylindrical rigidity, ν is the Poisson's ratio.

We shall define tangential (membranous) and flexural stresses by a formula:

$$\sigma_r^{(T)}=\frac{N_r}{h},\quad \sigma_r^{(N)}=\frac{\sigma M_r}{h^2}, \quad (27)$$

and maximum stress in the shell

$$\sigma_r^{(\max)}=\sigma_r^T+\sigma_r^N. \quad (28)$$

Particular cases

a) Let's consider the case where vacuum exists in side of the shell. Hence, it follows that $\mu'=0,\ \rho_0'=0$. Then

$$p_{rr}=p_{r\theta}=p=0. \quad (29)$$

The boundary conditions (18), (20) are transformed to

$$\frac{\partial U_r}{\partial t}=\frac{\partial\Phi}{\partial r},\quad q_r=\rho b,\quad q_0=0. \quad (30)$$

b) If inside the shell the ideal fluid is contained, then

$$\mu'=0.$$

Then

$$P_{rr}=-P,\quad P_{r\theta}=0,\quad P=-\rho_0'\frac{\partial^2\psi}{\partial t^2}. \quad (31)$$

Therefore, the problem is to be solved under the following boundary conditions:

$$\frac{\partial U_r}{\partial t}=\frac{\partial\Phi}{\partial r},\quad \dot{U}_r=v_r,\quad \dot{U}_\theta=0,\ q_r=P+P_b,\quad q_\theta=0. \quad (32)$$

Numerical computations and analysis

Numerical computations are carried out for a cylindrical shell, filled with several types of media and air is taken as an acoustical medium. In this case, the medium with water is taken as an acoustical medium and inside the shell the medium is air. In the second case, the acoustical medium is air; inside the shell is the medium in the form of water. Moreover, both cases are calculated at relatively low $\omega=200$, 250, 300 and high $\omega=3000$, 4000, 5000 frequencies.

Charts of dependence of tangential and flexural stresses from the angular coordinate θ at the values $\omega=250$ and $\omega=5000$ rad/s were obtained.

We shall note that stresses are defined by an expression

$$\sigma=(R+iJ)e^{-i\omega t}=\sqrt{R^2+J^2}l^{-i(\omega t-\gamma)},\ \gamma=actg\frac{J}{R},$$

Therefore

$$\sigma_\theta^{(T)}=(R_e\sigma_\theta^{(T)}+iJ_m\sigma_\theta^{(T)})e^{-i\omega t},$$

$$\sigma_\theta^{(N)}=(R_e\sigma_\theta^{(N)}+iJ_m\sigma_\theta^{(N)})e^{-i\omega t}.$$

Since the stress is the actual value, we shall define the maximum principal stress in the following way

$$\sigma_\theta^{(\max)}=\sqrt{\left[R_e\left(\sigma_\theta^{(T)}+\sigma_\theta^{(N)}\right)\right]^2+\left[J_m\left(\sigma_\theta^{(T)}+\sigma_\theta^{(N)}\right)\right]^2}.$$

Hence, the real parts of stresses correspond to the primary – instant $t=0$, and the imaginary parts to the instant of time through the quarter of the period $t=T/4$ where $T=2\pi/\omega$.

The data of the material of the shell are as follows: $R=10$ cm, $h=0.1$ cm, $E=2\cdot10^6$ kg/cm^2, $\nu=0.3$.

Fig. 1 and 2 show the dependences of tangential and flexural stresses on the angular coordinate at the action of acoustical waves on the cylindrical shell, filled with viscous compressed medium. Fig. 1 corresponds to the first case, and Fig. 2 to the second case, we shall note that in Fig. 1 (1–8) the curve 1 relates to air; on Fig. 2 (1–8) to water.

Conclusions

While analyzing the results it is possible make the following conclusions:

– if the acoustical medium is the medium of the type water, the maximum principal stress on the medial surface of the shell, created by the plane harmonic wave is much bigger then when the acoustical medium is air. This affirmation remains valid at relatively low and at high frequencies of vibrations;

– with the increase of the frequencies on a cylindrical shell the flexural stress is increasing;

– at a low frequency the flexural stress on the medial surface of shells is relatively low;

- density also has an influence on the stressed state of shells.

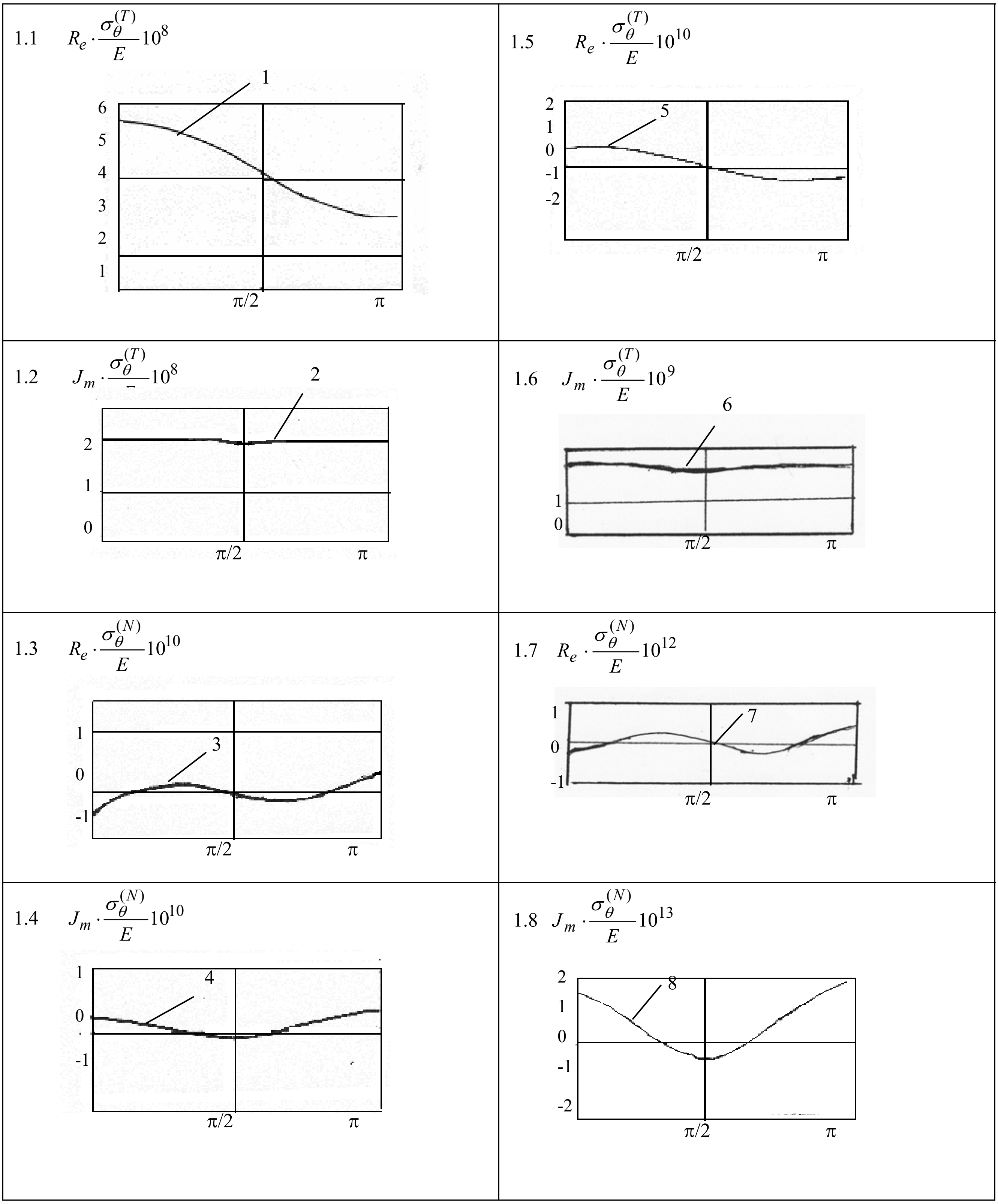
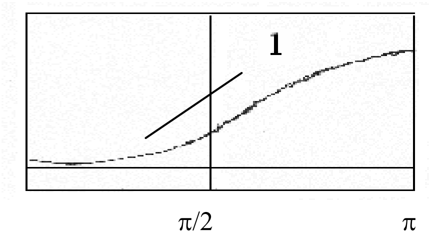
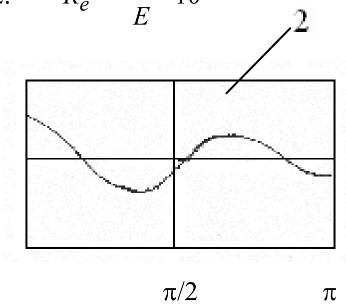


Fig. 1. **Dependence of tangential and bending stresses on angular coordinates θ at the action of an acoustical wave, with air present in the cylindrical shell**

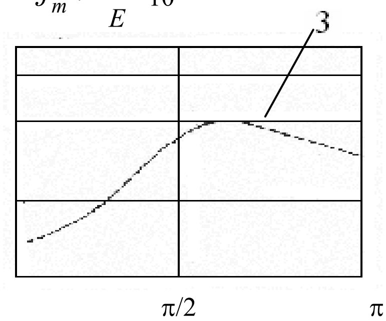
2.1. $R_e \cdot \frac{\sigma_\theta^{(T)}}{E} 10^{11}$



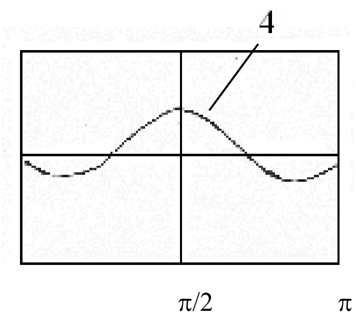
2.2. $R_e \cdot \frac{\sigma_\theta^{(N)}}{E} 10^{12}$



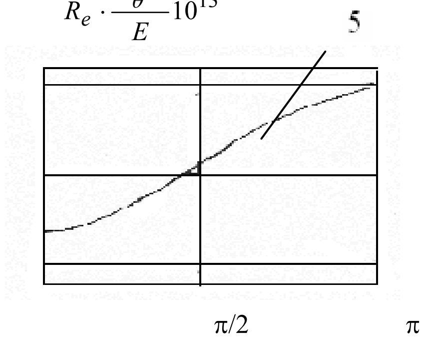
2.3. $J_m \cdot \frac{\sigma_\theta^{(T)}}{E} 10^{11}$



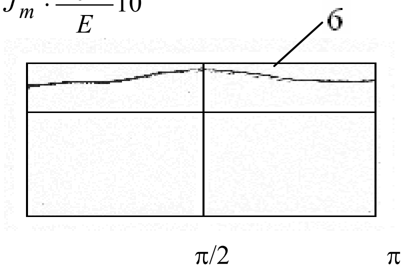
2.4. $J_m \cdot \frac{\sigma_\theta^{(N)}}{E} 10^{12}$



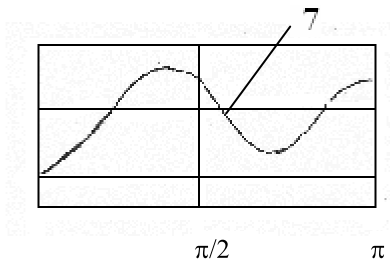
2.5. $R_e \cdot \frac{\sigma_\theta^{(T)}}{E} 10^{13}$



2.6. $J_m \cdot \frac{\sigma_\theta^{(T)}}{E} 10^{12}$



2.7. $R_e \cdot \frac{\sigma_\theta^{(N)}}{E} 10^{15}$



2.8. $J_m \cdot \frac{\sigma_\theta^{(N)}}{E} 10^{15}$

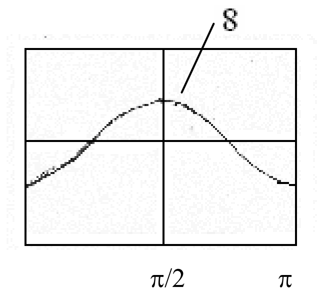


Fig. 2. **Dependence of tangential and bending stresses on angular coordinates θ at the action of an acoustical wave, with the cylindrical shell filled with water**

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D. Gužas

Akustinių bangų poveikis cilindriniam kevalui

Reziumė

Dabartiniai statiniai, pavyzdžiui, bokštai, vamzdynai ir kiti cilindrinės formos kevalai su skirtingomis akustinėmis terpėmis viduje ir išorėje, turi tikti naujoms technologijoms ir atitikti jų reikalavimus.

Nagrinėjamuoju atveju reikia žinoti, kaip akustinės bangos veikia cilindrinio kevalo paviršių. Šame darbe tyrinėjamos plokščios harmoninės bangos, sklindančios tam tikru apskritiminiu dažniu akustinėje terpėje, poveikis cilindrinės formos kevalui. Krintanti banga sužadina atspindžio ir sklaidos akustines bangas, kurios sužadina virpesius kevalo konstrukcijoje. Šis poveikis gali būti įvairus. Tyrimo rezultatai parodė, kad tangentinių ir lenkimo jėgų įtaka veikiant akustinėms bangoms priklauso nuo kampinių koordinačių θ. Esant žemiems dažniams, lenkimo jėgų poveikis cilindriniam paviršiui yra nedidelis. Didėjant dažniams, šių jėgų poveikis didėja. Akustinės bangos, sklindančios vandens terpėje, poveikis yra didesnis negu sklindančios dujose (ore) ir priklauso nuo terpės tankio.

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