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Transformation of ultrasonic wavefronts on the boundary liquid/solid

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Introduction

The motivation for this work was the need to know exact form of ultrasonic wavefronts in order to use this information in the ultrasound reflection tomography performing backprojection [1]. The purpose of the ultrasonic reflection tomography is to reconstruct from the data collected a quantitative cross-sectional image, which displays a distribution of the specific ultrasonic parameter in the material under a test. Usually the filtered backprojection method along circular arcs is used to obtain reconstructed images [1-3]. In order to achieve a better resolution of the images reconstructed in the ultrasound reflection tomography, the curvature of the wavefronts has to be known and taken into account performing backprojection.

Reconstruction of wavefronts of pulsed ultrasound fields is very complicated, because amplitude and phase of a wave are not constant and varies in all directions. J.P.Weight and et al. [4-6] extensively theoretically and experimentally studied the pulsed wave fields in fluids. These studies showed, that assuming idealized piston source, experimental results are in good agreement with theoretical results.

The propagation of ultrasonic beams in solids is even more complicated, because in solids exist several types of waves. Calculations of the field were made assuming solid to be isotropic and the experimental measurements of the field on the surface of the solid were carried out [7,8].

All mentioned works were devoted to calculation and experimental studies of the ultrasonic pressure amplitude beam profiles. To our knowledge, there are no works proposing methods for reconstruction of ultrasonic wavefronts. In fluids it’s not so difficult to measure wavefronts at various points. But it not the case in the solid media - we can not directly measure wavefronts in solids, so the only possibility is to calculate them from the measured parameters of the wave.

The goal of this work was to reconstruct wavefronts in solids, after wavefront transformation on the liquid/solid boundary. The dimensions of the transducer are taken into account during simulation. Calculations of the fields were performed in media consisting of two different materials, for example, water - steel. Ultrasonic wavefronts were calculated from measured parameters of the wave using the method, proposed by the authors [9,10]. Proposed method reconstructs coordinates of ultrasonic wavefronts from the measured time of flight. Measurements were performed using immersion technique. The side drilled hole was used as a reflector in the test block, and the time of flight was measured. From this data the shape of the wavefront is reconstructed. Therefore, using this method, it is possible to calculate wavefronts not only in fluids, but in solids also.

Impingement of an ultrasonic wave on a boundary between two media

When an ultrasonic wave encounters an interface between two media, the energy of the wave is partitioned in a manner that depends upon the type of the incident wave, upon how the wave approaches the interface, and upon the acoustic properties of the two media. In the simplest formulation of the problem, a plane ultrasonic wave propagating in a homogeneous media of uniform density ρ_1 and of constant sound speed c_1 is normally incident on a plane boundary separating the first media from a second homogeneous media of different acoustical properties represented by ρ_2 and c_2. A simple relationship, known as Snell’s law, describes the angle of refraction of transmitted wave. A common case, met in practice, which we also use in our modeling and experiments, is the water/steel interface [11].

According to [12] no phase shift occurs between the incident and the transmitted wave, regardless of which media has the higher acoustic impedance. In practice, the actual situations are often considerably more complicated than the ideal conditions described in theory.

The ultrasonic field simulation

According to various methods it can be shown that acoustic field of plane disk transducers consist of plane and edge waves. Such a presentation has been proved to be valid by many theoretical simulations and practical experiments [13-19]. It was shown that edge waves are concentrated on a geometrical axis of the transducer, which in the case of a circular transducer corresponds to equal distances from transducer edges. This line can be called an acoustical axis of the transducer. On the acoustical axis the signal in a far field corresponds to a maximum value in perpendicular to axes plane. In the case of two materials acoustical axis may not coincide with geometrical axis [20]. The geometrical axis in second media we shall determine like extension of geometrical axes only with the angle corresponding to the Snell's law. The acoustical axis we determine as positions with a maximal amplitude.

Usually the Snell's law is determined for plane waves, but under some approach it is valid for other shapes of waves too and is used in similar approaches [21]. Here we made an assumption that the Snell's law is valid for each ray emitted by any point of transducer. In this case, if we want to calculate the path of the waves for two selected points, one on the transducer surface and other in the second material, it is necessary to find the point satisfying the Snell's law at the refraction plane between two materials. Of course, such an approach does not take into account a transmission coefficient for a signal amplitude.

The described model can be extended for calculation of acoustical fields in the case of two, three and more materials. Of course such an approach, based on a scalar model, does not take into account all complexity of waves in solids, but it can be fast enough and show the structure of a transducer field.

We can see that a big part of an energy is concentrated in three pulses. The first pulse corresponds to the arrival time of a plane wave from the surface of a transducer. The second and the third pulses correspond to the arrival times of edges waves from the nearest and farthest edges of disk shape transducer. Outside the direct beam region they corresponds to the first and second pulses, because in this case a plane wave is absent.

Such a model usually can be used only for homogenous media. It is difficult to derive a similar analytical solution for the case of two or more materials, especially if a transducer is not parallel to the surface, because the model becomes not axisymmetrical.

For a solution of this problem we have made assumption that there must be no significant difference in the form of the pulse response and the main difference is only in the plane and edge waves arrival moments t_0, t_1, t_2. In this case for a calculation of the pulse response at the point $P(x,z)$ (Fig. 1) it is necessary:

1. To determine the times t_0, t_1, t_2;
2. To find the point P'(x',z') in a homogeneous media with specified ultrasound velocity, for which the pulse response has the same time difference $t_1^{'} - t_0^{'} = t_1 - t_0$ and $t_2^{'} - t_1^{'} = t_2 - t_1$, where $t_0^{'}, t_1^{'}, t_2^{'}$ are the arrival moments of the waves to the point $P'(x',z')$;
3. To calculate the pulse response of a transducer using the standard algorithm for a disk shape transducer. Of course, the absolute times must be corrected according real to the t_0 for the point $P(x,z)$.

The time of flights t_1 and t_2 can be calculated from system of equations, according to which each of the rays, emitted by points 1 and 2 of transducer plane must satisfy the Snell's law at the boundary of the second medium:

$$\begin{cases} c_1 \cdot r_{11} \cdot (x - x_{\mathrm{p1}}) = c_2 \cdot r_{12} \cdot (x_{\mathrm{p1}} - x_{\mathrm{k1}}), \\ c_1 \cdot r_{21} \cdot (x - x_{\mathrm{p2}}) = c_2 \cdot r_{22} \cdot (x_{\mathrm{p2}} - x_{\mathrm{k2}}), \end{cases} \quad (1)$$

$$t_1 = \frac{r_{11}}{c_1} + \frac{r_{12}}{c_2}, \quad (2)$$

$$t_2 = \frac{r_{21}}{c_1} + \frac{r_{22}}{c_2}, \quad (3)$$

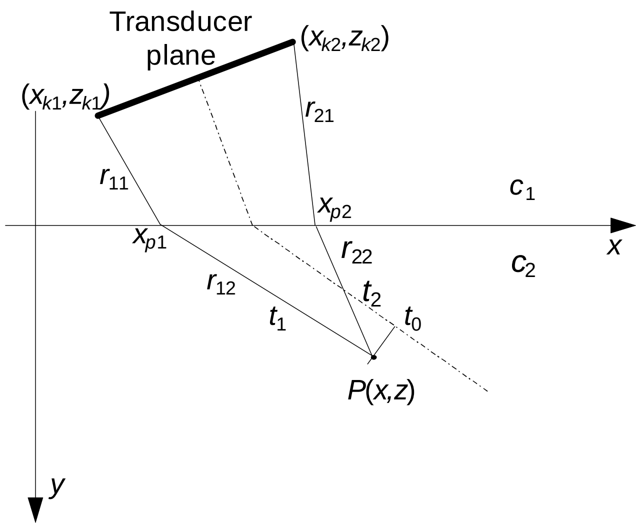

Fig. 1 **Field calculation for a circular shape transducer for the two material case**

where c_1, c_2 are the ultrasound velocities in both materials and r_{11}, r_{12}, r_{21}, r_{22}, x_{p1}, x_{p2}, x_{k1}, x_{k2} are explained in Fig. 1.

The time t_0 for a plane wave can be calculated as

$$t_0 = \frac{|z_{k0}|}{c_1 \cos(\alpha_1)} + \frac{z}{c_2 \cos(\alpha_2)} + \frac{(x - x_a)\sin(\alpha_1)}{c_1}, \quad (4)$$

where $z_{k0} = (z_{k1} + z_{k2})/2$, α_1 - is the central beam angle in the first and the second media, and

$$x_a = x_{k0} + z \cdot \mathrm{tg}(\alpha_2) + |z_{k0}| \cdot \mathrm{tg}(\alpha_1). \quad (5)$$

The purpose of the second step is to find some point in a homogeneous media (for example, with the ultrasound velocity c_1) with the same time difference t_2- t_1 like in the first step. The coordinates of this point can be found from the equations presented below:

$$\Delta = \sqrt{(d/2 + z')^2 + (x')^2} - \sqrt{(d/2 - z')^2 + (x')^2}, \quad (6)$$

where d is the transducer diameter;

We can find that

$$x' = \begin{cases} \sqrt{\dfrac{a_+^4 + a_-^4 + \Delta^4 - 2a_+^2\Delta^2 - 2a_-^2\Delta^2 - 2a_+^2 a_-^2}{4\Delta^2}}, & |\Delta| > 0, \\ \sqrt{t_1^2 c_1^2 - d^2/4}, & \Delta = 0, \end{cases} \quad (7)$$

where

$$\Delta = l_3 - l_2 = (t_3 - t_2)c_1,$$

$$a_+ = d/2 + z', \quad a_- = d/2 - z',$$

$$z' = (x - x_a)\cos(\alpha_1).$$

In the third step, when coordinates of the equivalent point $P'(x',z')$ are determined, it is possible to calculate the transducer pulse response using diffraction model for a circular shape transducer. However, this pulse does not take into account an absolute time and amplitude. In special cases both of them can be simply corrected.

After these three steps an acoustical field p_a (t,x,y,z) of the transducer with driving pulse $u(t)$ can be obtained using

$$p_a(t,x,y,z) = u(t) \otimes h_t(t,x,y,z), \qquad (8)$$

where ⊗ denotes convolution. Of course, in this approach the transmission coefficients for a signal amplitude were not taken into account.

Algorithm for calculation of wavefronts coordinates

The detailed explanation of the wavefront calculation algorithm is given in [9], and only brief description of it is included here.

A transmitter radiates an impulse into space, and the time of flight t_f of the impulse from transmitter to receiver is measured. Usually it is assumed that a wavefront is location of points with an equal phase. In the case of pulsed ultrasonic fields we can assume, that wavefront is location in space which is reached by the wave after the same time t_c. So, when the time of flight from a transmitter to a receiver t_f is measured, and the time of flight from the transmitter to the particular wavefront t_c is known, we can calculate the distance from the receiver to the wavefront:

$$a_{tf} = \left(t_f - t_c\right)c , \qquad (9)$$

where t_f is the measured time of flight; t_c is the time of flight when receiver is in front of the transmitter, c is the velocity of ultrasound in the medium.

The exact curvature of the wavefront is not known, so we don't known how to backproject calculated distance from the receiver to the wavefront. This distance can be backprojected in all directions along the radius of the circle, the center coordinates of which are coordinates of the receiver, and radius is equal to the distance from the receiver to the wavefront $a_{tf,}$ calculated according to(9).

Computation results

The field radiated by a circular transducer was calculated. Then, using above described methods, wavefronts at various distances from the transducer were reconstructed. The calculations were performed for a wideband disk type transducer of the radius R=1mm and with the center frequency f=2.5MHz. It was assumed, that transducer radiates burst with the Gaussian envelope, duration of which was two periods. The velocity of ultrasound was c_1=1.48 mm/μs in the first (fluid) media and c_2=5.7 mm/μs in the second (solid) media.

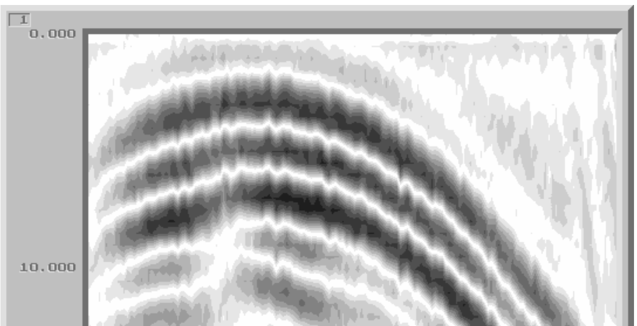

Fig. 3. **B-scan image of the transducer field.**

The transducer was scanned from x_{r1}=-7 mm to x_{r2}=7 mm, distance between transmitter and solid surface varied from 10 to 60 mm, and the distance from the solid surface to the reflector was 40 mm.

Ultrasonic beams, radiated by sources of finite dimensions, diverge and for this reason ideal plane waves can not be excited. This spreading reduces the intensity of the wave. Measurements, which we are performing, are carried out in a far field zone. According to our investigations [10], it means, that in a far field zone we can assume a transducer to be a point type, which transmits spherical waves.

In the case when wave propagates through the boundary of the two media with different velocities, it becomes quite complicated, how to find the corresponding radius of spherical wavefront. We assumed, that the radius of the wavefront for this case can be calculated using formula:

$$R = \frac{c_2 * a_2}{c_1} + a_1 \qquad (10)$$

where c_1 is sound velocity in the first media, c_2 is sound velocity in the second media, a_1 is the distance, which the wave propagates in the first media, and a_2 is the distance, which the wave propagates in the second media.

The time of flight was evaluated using the signal zero-crossing technique. It means, that the time of flight t_f was measured, after signal reaches maximum and then changes a sign from positive to negative.

Experimental results

Experimental measurements were carried out in order to test validity of the proposed methods. The block diagram of experimental set-up is presented in Fig. 2. The transducer and solid object were located in the water tank. The same transducer was used as transmitter and as receiver. The frequency of the transducer was f=2.5 MHz, and the beam divergence angle 38°. The signals were measured in reflection mode. The transducer was scanned from x_{r1}=0 mm to x_{r2}=15mm, the distance between the transmitter and the solid surface varied from 10 to 60mm, and the distance from the solid surface to the reflector was 40mm. 200 signals were acquisited, 105 sample in each. Data acquisition was done using the imaging system [22], which was developed at the ultrasound laboratory in Kaunas University of Technology.

In Fig. 3. B-scan image of the reconstructed field is presented. In Fig. 4 - 6 the wavefronts reconstructed at various distances from the transducer are presented.

The reconstructed wavefronts in the figures are marked as RW, the wavefronts, computed using the proposed formula, are denoted as CW, and the wavefronts, approximated as nearest circles, are marked as AW. The

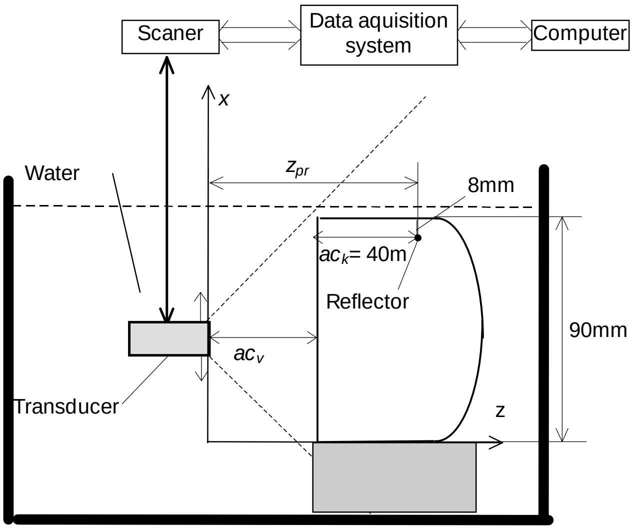

Fig. 2. **Experimental setup**

uncertainty between the reconstructed wavefront and the spherical wavefront in the figures is marked as RA, the uncertainty between the reconstructed wavefront and the calculated wavefront is marked as RC.

As can be seen from the results presented, the calculated wavefronts are in a good agreement with spherical wavefronts. So, with the uncertainty $RS < 0.1\lambda$, we can assume, that the transducer is a point type transducer and it is transmitting a spherical wave. Also from the results presented it follows, that the radius of the wavefront, after the wave propagates through the liquid/ solid boundary, we can calculate using proposed formula with small uncertainty. On the sides of the zone errors are bigger, because the amplitude of the signal is small and errors can be due to the fact, that zero crossings of the pulsed signal are not correctly found.

Conclusions

Using the proposed method wavefronts of ultrasonic waves in solid from the data collected at discrete points in a fixed plane, perpendicular to a propagation direction, has been reconstructed. The accuracy of the method was determined comparing the reconstructed wavefronts with those obtained theoretically. From the results presented it is seen that wavefronts, computed using the formula proposed are in a good agreement with the reconstructed wavefronts. The wavefronts remain spherical even after propagation through the liquid/ solid boundary, at least with perpendicular orientation of the transducer. The radius of this sphere can be found using proposed formula.

Acknowledgments

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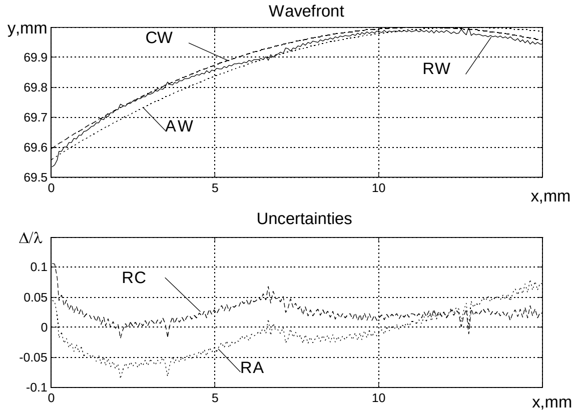

Fig. 4. **Wavefront reconstructed at the distance y=70 mm from the transducer and corresponding uncertainties**

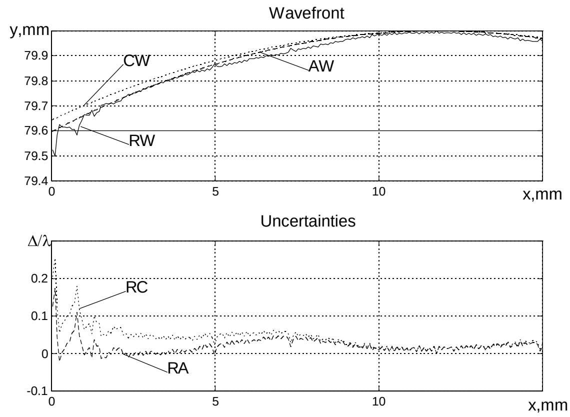

Fig.5. **Wavefront reconstructed at the distance y=80 mm from the transducer and corresponding uncertainties**

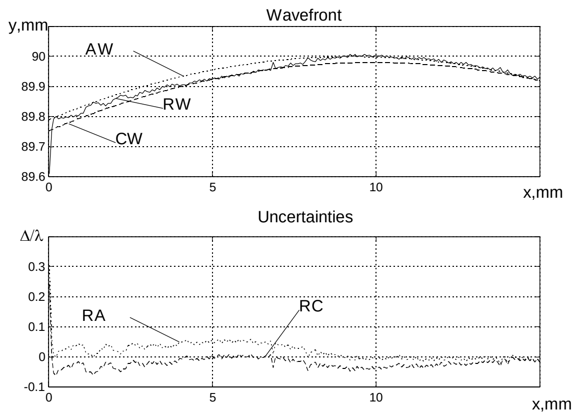

Fig. 6**. Wavefront reconstructed at the distance y=90 mm from the transducer and corresponding uncertainties**

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Ultragarso bangø frontø transformacija perëjus skysèio ir kieto kûno ribà

Reziume

Vienas ið ultragarsinës tomografijos kokybæ nusakanèiø veiksniø yra rekonstrukcijos kreiviø tikslumas. Kaip rekonstrukcijos kreivës ultragarsinëje atspindþio tomografijoje yra naudojami ultragarso bangø frontai tiriamojoje aplikoje. Ðiame straipsnyje analizuojamas ultragarso bangø frontø atkûrimas kietuose kûnuose. Kompiuterinis modeliavimas ir eksperimentiniai matavimai buvo atlikti imersiniu bûdu. Nagrinëjamuoju atveju keitiklio suformuoti bangø frontai transformuojasi, pereidami skysèio ir kieto kûno ribà. Naudojant plaèiakampá keitiklá, buvo tiriama transformuotos bangos fronto formos priklausomybë nuo atstumo tarp keitiklio ir kieto kûno pavirðiaus. Straipsnyje pateikti kompiuterinio modeliavimo ir eksperimentiniø matavimø metodikos ir rezultatai.

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