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Modelling techniques for ultrasonic wave propagation in solids: 2D case

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Introduction

The small wavelength elastic vibration often termed as ultrasonic are of widespread use today. It found wide implementation in non-destructive testing, geometric measurements, vibroageing processes. The ultrasonic measurements have won the strong position providing the reasonable amounts of information at comparatively low price in comparison with other alternative methods. By virtue, the ultrasonic measurements in solids and composite bodies of complicated geometrical shape often require to develop the case oriented equipment and image processing software basing upon the knowledge of the physical essence of the wave propagation in the environment under investigation. As powerful tools for modeling of the ultrasonic measurement process the finite element method (FEM) and/or boundary integral equation method (BIEM) can be used. The modeling of the ultrasonic wave propagation can be considered as the general small displacement transient structural dynamic problem the solution techniques of which has been perfectly described in the early 1980s by numerous researchers and implemented today in practically all the FEM software available on the market.

However, the practical application of FEM or BIEM is not always easy and efficient. As a rule, FEM requires highly refined meshes for proper representation of the strain and displacement field taking place in a body the dimensions of which are of considerably larger size than the length of the elastic wave propagating in a body. In this way even simple academic problems often require huge amounts of computational resource and for many practical applications the real price of finite element modeling is prohibitive. BIEM does not have such restrictions on element meshes but the computational time required is huge.

This work deals with the FEM approach by developing the mass matrix modification method already introduced in [5] trying to find ways for efficient numerical simulation of the above mentioned problems in 2D case. The efficiency of the method is understood as its speed and accuracy by using moderate computational resources. Solid body is considered as a body the material properties of which (Young's modulus, density, Poison's ratio) are close to metals. The wavelength of interest is defined basing on the ratio of the wavelength to dimensions of the body and might vary from the length comparable to the size of the body to the wavelength making only 1/1000 part of the body size.

Investigation

It was demonstrated by the authors [5] that the mass matrix modification techniques provide reasonable accuracy when modeling short wave propagation in rough meshes, e.g., including only 5 finite elements per wave length. The essence of the method is based on the dynamic reduction principles in order to find optimum mass distribution law between the nodes of an element.. The mass matrices are modified, involving the transformation of equations of a finite element into modal coordinates and changing the weighting factors of the modal contributions to the total response.

Consider the dynamic equation of a finite element with damping neglected as

$$[M]\{\ddot{U}\}+[K]\{U\}=\{F\};\ (1)$$

where $[M]$, $[K]$ are the mass and stiffness matrices, $\{U\}$, $\{F\}$ - displacement and external load vectors.

Equation (1) can be transformed into modal coordinates by obtaining the eigenfrequencies and eigenforms of the element from the equation

$$\det\left|[K]-\omega^2[M]\right|=0\,; \qquad (2)$$

Equation (1) presented in modal coordinates reads as

$$[I]\{\ddot{Z}\}+diag(\omega^2)\{Z\}=[Y]^T\{F\}\,; \qquad (3)$$

where $\{Z\}$ - vector of the generalized displacements of the element related to the displacement vector $\{U\}$ as $\{U\}=[Y]\{Z\}$, $[Y]$ - matrix of normalized eigenvectors satisfying the relation $[Y]^T[M][Y]=[I]$.

Now we truncate the dynamic contributions or change of weighting factors defining the extent of participation of selected modal contributions to the total response of the element.

The matrix multipliers are defined as

$$[A_1]=\begin{bmatrix} c_1 & & & \\ & 0 & & \\ & & \dots & \\ & & & 0 \end{bmatrix}_{n*n},\ [A_2]=\begin{bmatrix} 0 & & & \\ & c_2 & & \\ & & \dots & \\ & & & 0 \end{bmatrix}_{n*n},$$

$$[A_n]=\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \dots & \\ & & & c_n \end{bmatrix}_{n*n}\,; \qquad (4)$$

where $\boldsymbol{n}$ is the number of degrees of freedom of the finite element, $\boldsymbol{c_i}$ - coefficient located in the position corresponding to the mode number which weighting factor is to be changed for obtaining the modified mass matrix.

To return to the original displacements the summation is carried out :

$$\left(\sum_{i=1}^{n}[M][Y][A_i][Y^T][M]\right)\{\ddot{U}\}+$$
$$+\left([M][Y]diag(\omega^2)[Y^T][M]\right)\{U\}=[M][Y][Y^T]\{F\}; \qquad (5)$$

First term in the equation (5) corresponds to the modified mass matrix, second - to the stiffness matrix of a finite element. The global stiffness and mass matrices of the structure now can be assembled from the obtained element matrices as usual.

The coefficients of the matrix multipliers should be chosen with care in order to ensure the total mass of finite element unchanged.

Following examples illustrate the mass matrix modification techniques applied to the elastic triangular simplex finite element. The steel quadrilateral plate has been set as an object under investigation. The dimensions of the plate are were set basing on the wavelength of the 2 MHz frequency (λ=3 mm) and are of 1.5 wavelength in length and 3 wavelengths in width. The plate was meshed by 200 right-angled triangle finite elements (Fig 1). The action of the piezoelectric transducer is presented lumped forces.

Each triangle simplex finite element has 6 degrees of freedom, thus the dynamic equations in modal coordinates are of sixth order. Consequently, here we have more space for selecting weighting factors for modal contributions to the total response of the element. However, such a treatment of the problem is more complicated as it was in uni-dimensional considered in [5]. It appears as reasonable to deal only with two groups of modes. First group includes first 3 rigid body modes of an element, and the second group the remaining 3 deformation modes. If we would consider the random change of weighting factors without the separation into the above mentioned groups of modes, the clear physical interpretation is hard to find. Moreover, it was obtained that the rigid body modes control the total mass of the finite element. Thus the values of the coefficients c_1, c_2 and c_3 should be set to unity.

The modified mass matrix analysis has been carried out in order to determine the contribution of the deformation modes. Two types of the modified mass matrix were calculated by using two sets of coefficients :

coefficient set N.1 {1,1,1,0,0,0}

coefficient set N.2 {0,0,0,1,1,1}

The modified mass matrices with the contribution of the rigid body modes (coefficient set N.1) and of the deformation modes (coefficient set N.2) are presented as follows:

$$[M_l]=\frac{\rho hS}{3}\begin{bmatrix}0.42 & -0.04 & 0.30 & -0.04 & 0.30 & 0.08\\ -0.04 & 0.35 & 0.02 & 0.35 & 0.02 & 0.30\\ 0.30 & 0.02 & 0.35 & 0.02 & 0.35 & -0.04\\ -0.04 & 0.35 & 0.02 & 0.35 & 0.02 & 0.30\\ 0.30 & 0.02 & 0.35 & 0.02 & 0.35 & -0.04\\ 0.08 & 0.30 & -0.04 & 0.30 & -0.04 & 0.42\end{bmatrix}$$

$$[M_h]=\frac{\rho hS}{3}\begin{bmatrix}0.08 & 0.04 & -0.04 & 0.04 & -0.04 & -0.08\\ 0.04 & 0.14 & -0.02 & -0.10 & -0.02 & -0.04\\ -0.04 & -0.02 & 0.14 & -0.02 & -0.10 & 0.04\\ 0.04 & -0.10 & -0.02 & 0.14 & -0.02 & -0.04\\ -0.04 & -0.02 & -0.10 & -0.02 & 0.14 & 0.04\\ -0.08 & -0.04 & 0.04 & -0.04 & 0.04 & 0.08\end{bmatrix}$$

The sum of the matrices [M_l] and [M_h] provides the well known finite element consistent mass matrix as

$$[M_l]+[M_h]=\frac{\rho hS}{3}\begin{bmatrix}0.50 & 0 & 0.25 & 0 & 0.25 & 0\\ 0 & 0.50 & 0 & 0.25 & 0 & 0.25\\ 0.25 & 0 & 0.50 & 0 & 0.25 & 0\\ 0 & 0.25 & 0 & 0.50 & 0 & 0.25\\ 0.25 & 0 & 0.25 & 0 & 0.50 & 0\\ 0 & 0.25 & 0 & 0.25 & 0 & 0.50\end{bmatrix}$$

The contents of matrix $[M_l]$ containing the highest values in its main diagonal are close to the contents of the lumped mass matrix. In matrix $[M_l]$ the sum of elements in rows and columns always equals unity. The sum of the elements in rows and columns of matrix $[M_h]$ always equals zero. Consequently, the coefficients in set N.2 could be set to any number as the total mass provided by $[M_h]$ is of zero value

Various sets of coefficients were used in order to find the optimum set to simulate the short length wave propagation in the rough finite element mesh containing 5 finite elements per wavelength. Fig. 2 represents the exact solution when 25 finite elements per wavelength were used.

In Fig.3 the results for the coefficient set (1,1,1,1,1,1) and consistent mass matrix containing 5 FE per wavelength are presented.

The wave pulse tends to move unsymmetrically leaning to the left. This deformation of the shape of the wave is a consequence of the right-angled triangle mesh applied to the structure. Under the application of such a

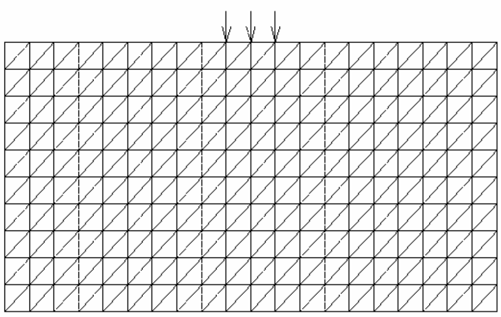
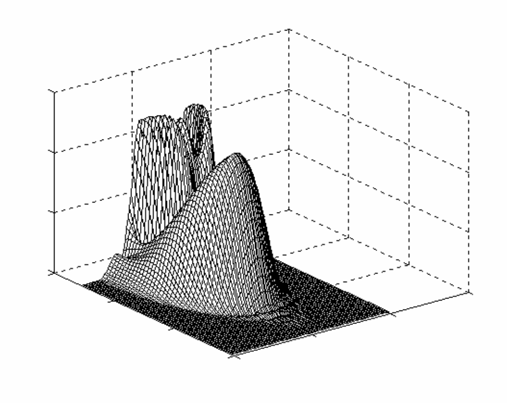


Fig.1. **The finite element model.**



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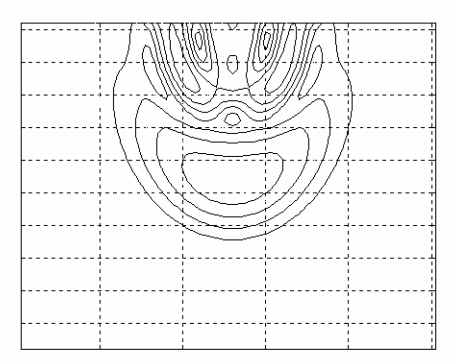

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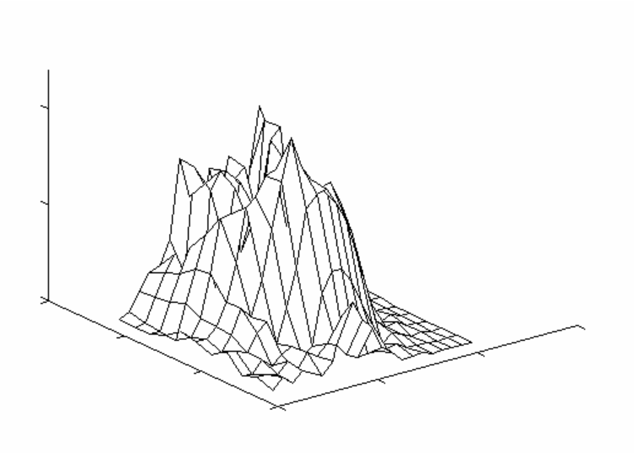
Fig.2. **Landscape (a) and contour (b) plot of the wave pulse for consistent mass matrix; 25 FE per wavelength.**

mesh the resulting stiffness of the plate differs slightly in two perpendicular directions.



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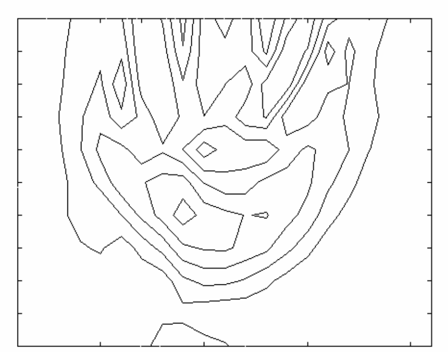

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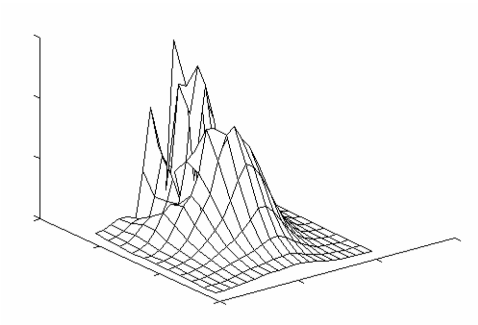
Fig.3. **Landscape (a) and contour (b) plot of the wave pulse for consistent mass matrix; 5 FE per wavelength.**



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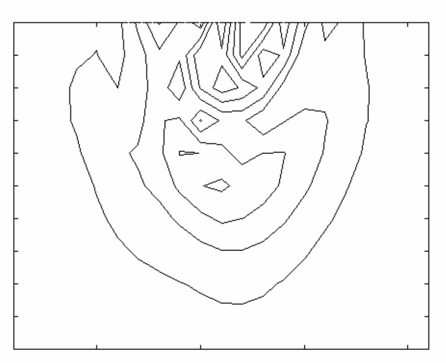

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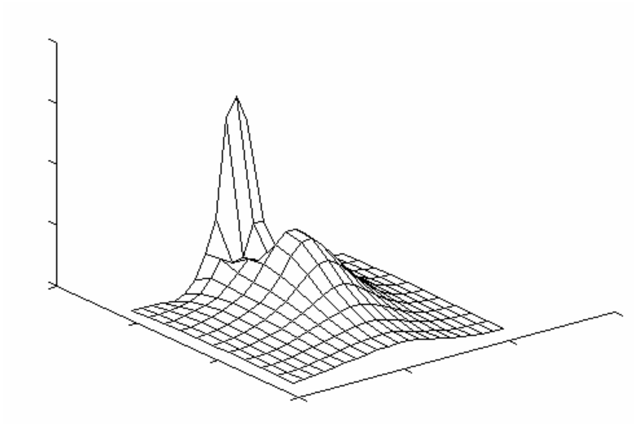
Fig.4. **Landscape (a) and contour (b) plot of the wave pulse by using modified mass matrices; 5 FE per wavelength; coefficient set (1,1,1,10,10,10) .**



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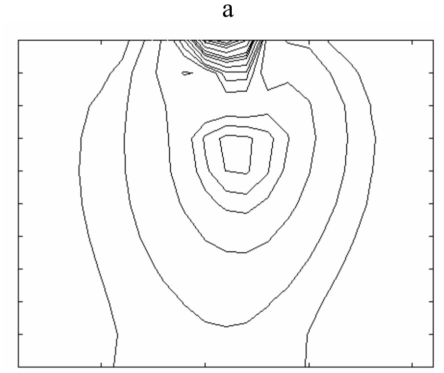

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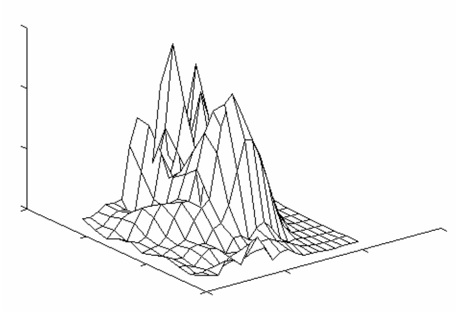
Fig.5. **Landscape (a) and contour (b) plot of the wave pulse by using modified mass matrices; 5 FE per wavelength; coefficient set (1,1,1,50,50,50) .**



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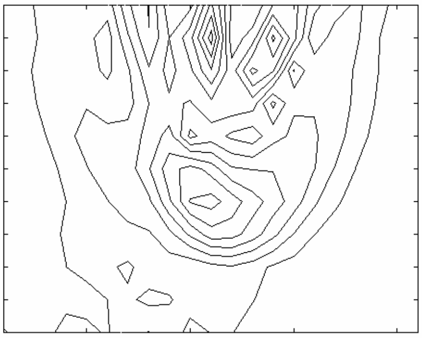

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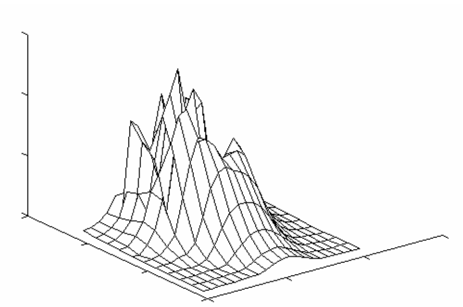
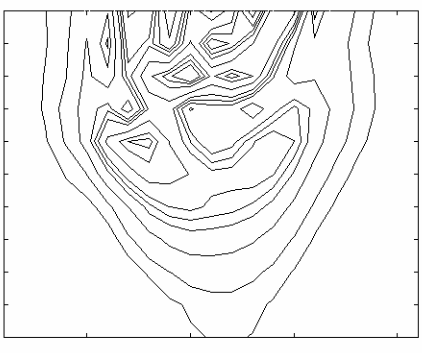
Fig.6. **Landscape (a) and contour (b) plot of the wave pulse; modified mass matrix; 5 FE per wavelength; coefficient set (1,1,1,10,1,1).**

The wave pulse may be corrected by increasing the weighting factors of deformation mode contributions. The set of coefficients (1,1,1,10,10,10) provides more reasonable results (Fig.4), however the slope of the wave front becomes less steep:

By increasing the weighting factors of the deformation mode contributions, the front of the wave pulse is being



a



b

Fig.7. **Landscape (a) and contour (b) plot of the wave pulse; modified mass matrix; 5 FE per wavelength; coefficient set (1,1,1,1,10,10) .**

distorted and the wave moves faster (Fig.5) :

The form of the wave pulse may be controlled in numerous ways by distributing the weighting factors among the deformation modes in different ways the. Fig. 6 represents the contour plot of the wave leaning to the left (coefficient set (1,1,1,10,1,1)), and Fig.7 - the hastening wave (coefficient set (1,1,1,1,10,10)).

To our opinion, making experiments in such a way can provide the optimal coefficient set.

Concluding remarks

In 1D case when structure is meshed by uniform finite elements the amount of computations in preparation stage can be saved, because the modal analysis for a single 1D element can be carried out analytically. In 2D case the eigenfrequencies and eigenforms should be obtained numerically for each individual element. Consequently, the considerable savings of computational resource can not be expected in 2D case. Despite of this the advantage of modified mass matrix method in 2D case is obtained because of the decrease of the finite element number that is expressed to greater extent in comparison with the 1D case. E.g. the finite element number decreases up to 25 times in 2D case to 5 times in 1D case. More precise results in modified mass matrix method should be obtained when using the uniform meshes of equilateral triangles.

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References

1. **J. H. Argyris, J. St. Doltsinis, W. C. Knudson, L. E. Vaz, K. J Willam.** Numerical Solution of Transient Nonlinear Problems, Comp. Meths. Appl. Mech. Engng., 17/18 (1979), pp 341-409;
2. **K. J. Bathe, E. L. Wilson.** Numerical methids in finite element analysis. - PRENTICE-HALL, INC., Englewood Cliffs, New Jersey;
3. **T. Belytschko**. On difficulty levels in non linear finite element analysis of solids. Bulletin for the international assocation for computational mechanics. 1996, nr 2;
4. **Y. Kawaga, G. M. L. Gladwell**. Finite Element Analysis of Flexure-Type Vibrators with Electrostrictive transducers, IEEE Trans. on Sonics and Ultrasonics, SU-17(1)(1970),pp 41-50;
5. **V. Daniulaitis, R. Barauskas**, Modeling Tecniques for Ultrasonic Wave Propagation in Solids., Ultragarsas, N1(29),1998;
6. **Й. Крауткремер, Г. Крауткремер** Ультразвуковой контроль материалов. Москва: Металлургия, 1991, 750 с.

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Ultragarsinių bangų sklidimo kietuose kūnuose modeliavimo metodai

Reziumė

Darbe nagrinėjami standžiame kūne sklindančių bangų, kurių ilgis gerokai mažesnis už kūno matmenis, kompiuterinio modeliavimo metodai. Tradicinis baigtinių elementų metodas tinka banginiams procesams tirti, jeigu bangos ilgis yra tos pačios eilės kaip ir konstrukcijos matmenys. Trumpėjant bangai, sprendinys vis labiau iškraipomas, kadangi diskretinis modelis nepakankamai tiksliai aprašo aukštąsias virpesių harmonikas. Darbe toliau plėtojamas ankstesniame darbe pasiūlytas baigtinio elemento masių matricos modifikavimo metodas trumposioms bangoms tirti.

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