

## Surface acoustic wave existence in anisotropic media A review

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### Introduction

The problem of the existence of the surface acoustic waves (SAW) in anisotropic solids has unsettled many experienced scientists for years and caused them to publish many papers on the subject. Some of them tried to answer the question whether SAW can propagate on an arbitrary cut of any anisotropic solid in any direction or there exists exceptions employing the surface impedance tensor first introduced by Ingebrigtsen and Tønning in 1969 [1-4]. In some others an effort has been made to employ the theory of uniformly moving dislocations [5, 6]. In this paper, we shall give a short review of the SAW existence considerations based upon the notion of the surface impedance tensor as more applicable and shall try to reveal the current state of what is known in this field.

### Acoustic waves in anisotropic solids

We shall start with the equation of motion for particles of homogeneous media:

$$c_{ijkl} \frac{\partial^2 u_k}{\partial x_i \partial x_l} = \rho \frac{\partial^2 u_j}{\partial t^2}, \quad j=1\dots 3, \quad (1)$$

where  $\mathbf{x}$  is the position vector,  $t$  is the time,  $\mathbf{u}$  is the particle displacement from its equilibrium,  $\rho$  is the material density, and  $c_{ijkl}$  is the elastic stiffness tensor, which is always positive definite [1]. Positive definiteness of the stiffness tensor expresses the fact that any displacement of body particles from their equilibrium results in increasing the potential energy of the body. The Einstein convention on the summation over repeated indices (both Latin and Greek) is accepted here and further, unless explicitly indicated otherwise.

Now let us consider an orthogonal pair of real unit vectors  $\mathbf{m}$  and  $\mathbf{n}$ . Stroh in 1962 has shown that for any function  $f$  with continuous derivative it is possible to find out such a vector  $\mathbf{A}$  and a scalar  $p$  that the displacement field described by it satisfies the equation of motion (1) [1]. Without loss of generality, we may assume that the function is an exponent, and the displacement field is

$$u_k = A_k f(\mathbf{m}\mathbf{x} + p\mathbf{n}\mathbf{x} - vt) = A_k e^{i\frac{\omega}{v}(\mathbf{m}\mathbf{x} + p\mathbf{n}\mathbf{x} - vt)}, \quad (2)$$

$j=1\dots 3,$

where  $v$  is a parameter with the dimension of velocity,  $p$  is a dimensionless parameter, and  $\omega$  is the frequency.

Substitution of Eq.2 into Eq.1 yields a set of linear algebraic equations with respect to  $A_k$ :

$$\{c_{ijkl}(m_i + pn_i)(m_l + pn_l) - \rho v^2 \delta_{jk}\} A_k = 0, \quad (3)$$

$j=1\dots 3,$

where  $\delta_{jk}$  is a Kronecker's delta symbol. As it is known, a homogeneous set of linear algebraic equations has a

nontrivial solution if the determinant of the matrix made up by the equation coefficients is zero, i.e.,

$$\det(D_{jk}) = \det\{c_{ijkl}(m_i + pn_i)(m_l + pn_l) - \rho v^2 \delta_{jk}\} = 0. \quad (4)$$

Assuming  $p=0$ , we obtain a third power equation with respect to  $v^2$ . This equation always has three positive roots. Substituting them into Eq.2, we obtain three particle displacement eigenvectors  $\mathbf{A}$ . The vectors so far are real, as the equation coefficients are real. Those vectors are the amplitudes of the three bulk waves, so far arbitrarily normalized, which displacement is uniform on any plane, perpendicular to  $\mathbf{m}$ , which is the direction of propagation of an acoustic wave. Generally, the direction of the bulk wave amplitudes  $\mathbf{A}$  neither coincides with  $\mathbf{m}$  nor is perpendicular to it. The propagation directions in which two of the bulk wave amplitudes are perpendicular, and the third which is parallel to  $\mathbf{m}$ , are called the pure mode directions [9] (however, sometimes the term is used for directions in which the wavefront normal  $\mathbf{m}$  coincides with the energy flow direction). The first two waves are transversal, the third one is longitudinal. Generally, the bulk waves in anisotropic solids are quasitransversal and quasilongitudinal. As their amplitudes are real, they are always linearly polarized. Moreover, due to the symmetricity of the tensor  $X_{jk}=c_{ijkl}m_i m_l$ , the polarizations of the three bulk waves are mutually orthogonal.

The three values of  $v$  obtained are the phase velocities of the quasitransversal and quasilongitudinal acoustic waves. Taking every propagation direction  $\mathbf{m}$  and computing the three velocities at each direction, we would obtain the three surfaces of the bulk wave velocities. In isotropic media every surface is a sphere. The two spheres of transversal wave velocities coincide, the sphere of longitudinal wave velocities is outside the spheres pertaining to transversal waves. Every direction in an isotropic solid is a pure mode direction. The situation is more sophisticated in anisotropic solids: generally, the surfaces are of different shapes. The directions in which some of the velocities coincide are called acoustic axes. Acoustic axes may or may not be associated with elements of symmetry; even triclinic crystals have acoustic axes. In the most cases quasilongitudinal waves are faster than quasitransversal ones. Eq.3 is invariant of the sign of the velocity. This implies that every surface of bulk wave velocities is centrosymmetric.

Even more interesting are the slowness surfaces (Fig. 1). Slowness is defined as a magnitude, inverse to velocity:  $c=1/v$ . One of the most important properties of a slowness surface is that the direction of its surface normal at any point coincides with the energy flow direction of the wave propagating from the beginning of the reference frame to the point selected.

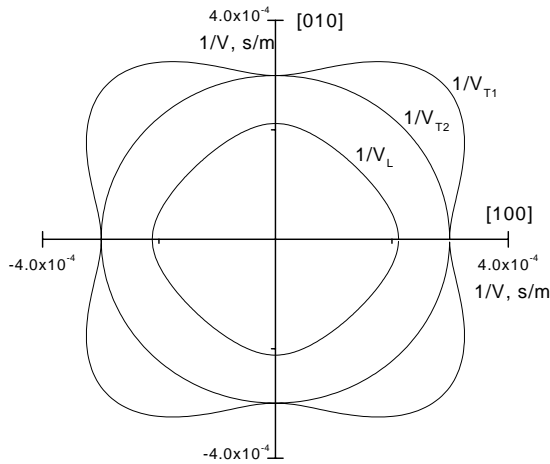


Fig. 1. The section of the bulk wave slowness for GaAs by the plane (0,0,1).  $V_{T1}$  and  $V_{T2}$  are the velocities of quasitransversal waves,  $V_L$  is the velocity of the quasilongitudinal wave

**Transonic states and limiting waves**

Consider a section of a slowness surface in the plane  $\mathbf{m}, \mathbf{n}$ . Let's choose a certain value of velocity  $v$  and put a point at the distance  $C=1/v$  from the centre of the reference frame on the axis of the propagation direction  $\mathbf{m}$  and draw a vertical line of constant slowness in the direction of  $\mathbf{m}$  (Fig. 2). Each point of the line corresponds to a hypothetical bulk wave propagating at the angle  $\alpha$  to  $\mathbf{m}$  with the slowness  $C_\alpha = C/\cos(\alpha)$ , or velocity  $v_\alpha = v\cos(\alpha)$ . On the other hand, the wave can be described in the reference frame  $\mathbf{m}, \mathbf{n}$  as  $\mathbf{u} = \mathbf{A}\exp(i(\omega/v)(\mathbf{m}\mathbf{x} + p\mathbf{n}\mathbf{x} - vt))$ , where  $p = \tan(\alpha)$ . Thus, the point of intersection between a wavefront and a line colinear with  $\mathbf{m}$  travels along this direction with the velocity  $v$ , larger than the actual phase velocity  $v_\alpha$  by  $1/\cos(\alpha)$ . Now the physical sense of the vertical line of constant slowness in the direction  $\mathbf{m}$  is clear: a wavefront of any bulk wave propagating at the angle  $\alpha$  to  $\mathbf{m}$  with the slowness  $C_\alpha = C/\cos(\alpha)$  travels along  $\mathbf{m}$  with the same slowness  $c$ . In the space of velocity the

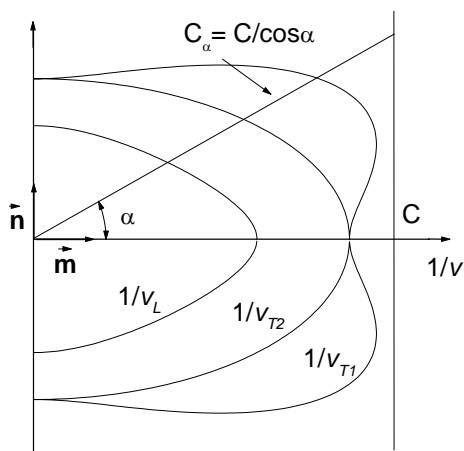


Fig. 2. The slowness curves and a line of constant slowness in the  $\mathbf{m}$  direction

vertical line of constant slowness in the direction  $\mathbf{m}$  is transformed into a circle with the radius  $R=v/2=1/(2C)$ , centered at the midpoint between 0 and  $v$ .

Now let's consider a problem inverse to searching for the velocities of bulk waves: let's choose a value of  $v$  and solve Eq.4 with respect to  $p$ . As Eq.4 is a sixth-order (sextic) algebraic equation with respect to  $p$ , it has six roots. Complex roots come in complex-conjugate pairs, because equation coefficients are real. Some of the roots may coincide. Let's draw a vertical line of constant slowness. If the velocity chosen is so small that the whole line is outside the outermost slowness surface, there is no such real  $p$  that the wave of the form (2) with the chosen  $v$  could propagate. Or, in other words, wavefronts of all the bulk waves that could propagate in the plane  $\mathbf{m}, \mathbf{n}$  travel along  $\mathbf{m}$  with a higher velocity than the chosen  $v$ . All the six roots of the sextic are complex conjugate pairs. Such a state and the velocity are called subsonic. The energy transfer direction for a subsonic wave is always perpendicular to  $\mathbf{n}$ . This can be merely deduced from the energy conservation law, taking into account that Eq.1, as an equation of motion, based upon the principles of Newtonian mechanics, consistent with the energy conservation law, can only have solutions satisfying it.

A state described by a line which is tangent to one or more slowness curves at one or more points is called a transonic state. Generally, there can be three transonic states, as there are three slowness curves, however, some of them may coincide. The notion of a transonic state is crucial to understanding the problem of surface wave existence. However, only the first transonic state with corresponding velocity  $v_L$ , called the limiting velocity, is of great importance. Therefore, when speaking of a transonic state, if not specified explicitly otherwise, we shall mean the first one. According to at how many points the line touches one or more slowness surfaces, transonic states are classified into six types (Fig. 3). Obviously, if a transonic state is of type 3, 5, or 6, it is the single possible transonic state for a given orientation, because all the three slowness surfaces are touched at once. Transonic states depend not only on the material properties, but also on the directions of  $\mathbf{m}$  and  $\mathbf{n}$  chosen.

When the velocity reaches its limiting value  $v_L$ , one or more pairs of complex conjugate roots coalesce into one or more pairs of equal real roots. A bulk wave described by such a real root is called a limiting wave [3]. As seen from the graphs, a type 1 transonic state has only one limiting wave, type 2 and 4 transonic states have two limiting waves, type 3, 5, and 6 transonic states have three limiting waves.

Finally, if the slowness line intersects with one or more slowness surfaces, such a state as well as the velocity are called supersonic.

By the way, examining of Eq.4 reveals that the slowness curves, that may be obtained by solving the equation with respect to  $v$ , cannot take any shape. The variety of possible shapes is restricted by the requirement that the three slowness surfaces altogether should have no more than six points of coincidence with any straight line. In the case of degeneracy, when two or three slowness surfaces coincide

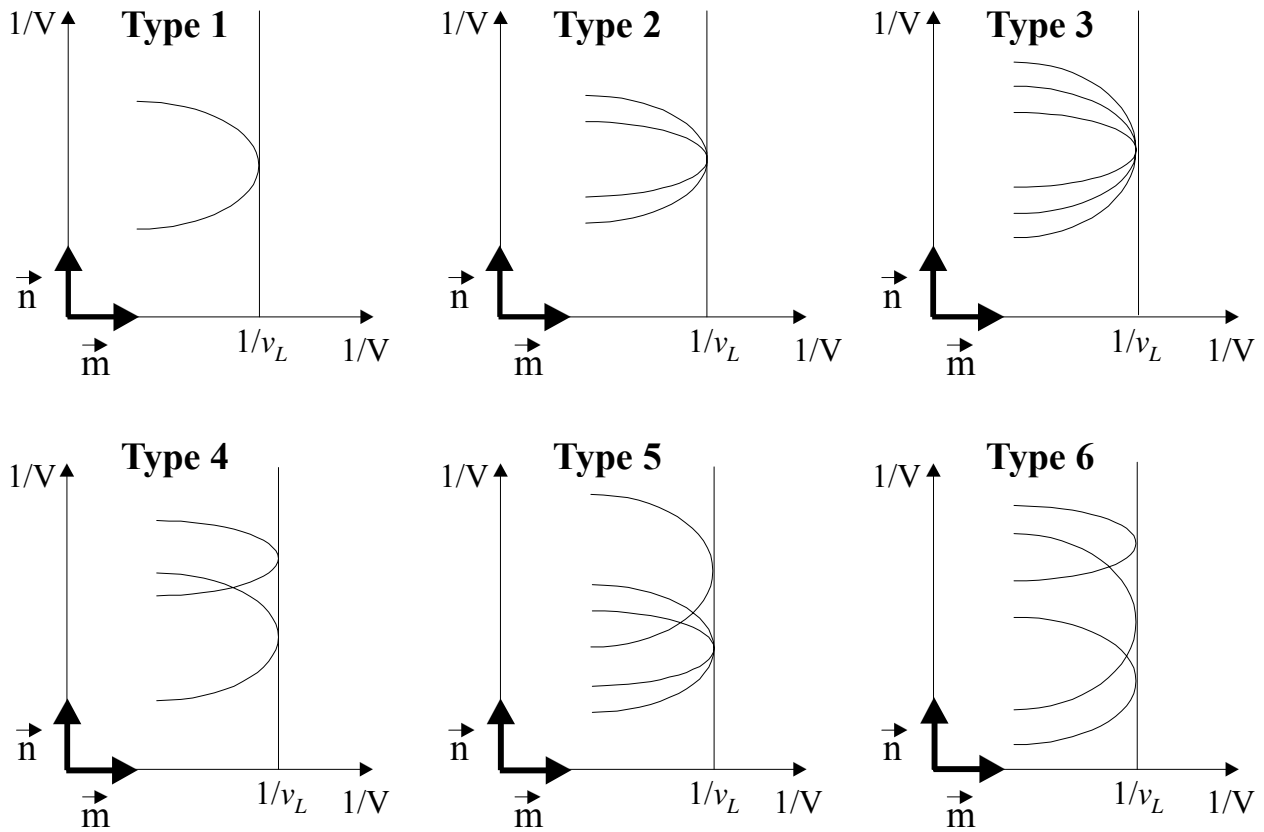


Fig. 3. The six types of transonic states.

at some single point, a solution of the form Eq.2 is no more the general solution of Eq.1. To obtain the general solution, one must include a solution of the form  $\mathbf{A}(\mathbf{B}, \mathbf{x}) \exp(i(\omega/v)(\mathbf{m}\mathbf{x} + p\mathbf{n}\mathbf{x} - vt))$  or with  $(\mathbf{B}, \mathbf{x})$  in a higher degree, where  $\mathbf{B}$  is a vector to be found. However, even having substituted this form into Eq.1, one couldn't obtain an equation of a higher degree than 6 with respect to  $p$ . Thus, in the contrary to what is stated in [4, 8], a slowness surface cannot take a shape of zero curvature, as in this case it would be possible to draw a line having an infinite number of points of coincidence with the flat region of the surface. This would mean that it is possible to find such a velocity  $v$  and such directions for  $\mathbf{m}$  and  $\mathbf{n}$  that the sextic Eq.4 has an infinite number of roots  $p$ . The slowness surface shown in Fig. 4.3 of [4] has only a seemingly flat region. Having calculated the slowness with double precision, one may see that it isn't flat.

### Exceptional limiting waves

Let us consider a reference frame in which  $\mathbf{x}_1$  coincides with  $\mathbf{m}$  and  $\mathbf{x}_2$  coincides with  $\mathbf{n}$ . In this reference frame  $\mathbf{m}=(1,0,0)$  and  $\mathbf{n}=(0,1,0)$ . The traction on a plane perpendicular to  $\mathbf{n}$  is

$$T_{2j} = c_{2jkl} \frac{\partial u_k}{\partial x_l} =$$

$$= c_{2jkl} \frac{\omega}{v} (m_l + pn_l) A_k e^{i\frac{\omega}{v}(\mathbf{m}\mathbf{x} + p\mathbf{n}\mathbf{x} - vt)} = \quad (5)$$

$$i\frac{\omega}{v} (c_{2jk1} + c_{2jk2}p) A_k e^{i\frac{\omega}{v}(x_1 + px_2 - vt)}, \quad j = 1...3.$$

If a limiting wave evokes no traction on planes perpendicular to  $\mathbf{n}=\mathbf{x}_2$ , it is called an exceptional limiting wave. Otherwise the limiting wave is called normal. Similarly, if all limiting waves of a transonic state are exceptional, the transonic state is called exceptional [1]. If at least one limiting wave is normal, the transonic state is normal. Naturally, an exceptional limiting wave, as satisfying the free surface condition  $\mathbf{T}_{2j}=0$ , can propagate in a half space with a boundary, perpendicular to  $\mathbf{n}=\mathbf{x}_2$ . It is why exceptional bulk waves are called improper or bulk acoustic waves. Yet there is another term - surface skimming bulk wave (SSBW). As shown in [1], the polarization of an exceptional limiting wave is always perpendicular to  $\mathbf{n}=\mathbf{x}_2$ . As indicate many of the published works, the *first* transonic state can be exceptional only if it is of type 1 (subsequent transonic states of some other types may also be exceptional; however, our interest regards only the first transonic state).

Generally, if a bulk wave of the geometry  $\mathbf{m}, \mathbf{n}$  with a certain  $p$  produces no traction on planes, perpendicular to  $\mathbf{n}$ , it is called exceptional bulk wave with respect to  $\mathbf{n}$ . Thus exceptional limiting waves is a subset of a broader class of exceptional bulk waves [3].

## Ingebrigtsen and Tonnings's formalism and the surface impedance tensor

Here we shall present an explicit derivation of the surface impedance tensor introduced by Ingebrigtsen and Tonnig in 1969 [9]. Let us consider a subsonic velocity  $v$ . Having solved the sextic, we acquire six values of  $p$ . Now we are able to choose three values out of the six in 20 ways. However, Ingebrigtsen and Tonnig proposed to choose the three values with positive imaginary parts, describing three waves of a particle displacement decaying with increasing  $x_2$ . As the three eigenvectors of the particle displacement  $\mathbf{A}_1$ ,  $\mathbf{A}_2$ , and  $\mathbf{A}_3$ , normalization of which is arbitrary, are linearly independent at  $v_L$  and below it [2], any other displacement vector  $\mathbf{X}$  with real or complex components may be expressed as a linear combination of those three with coefficients  $\xi_i$ :  $\mathbf{X} = \xi_\alpha \mathbf{A}_\alpha$ . Treating the triad of  $\xi_\alpha$  as a column vector, and the displacement eigenvectors as a matrix  $A_{i\alpha}$ , a column of which describes the three components of an eigenvector  $\mathbf{A}_\alpha$ , the latter expression may be written as  $X_i = A_{i\alpha} \xi_\alpha$ , or in the matrix form,  $\mathbf{X} = \mathbf{A} \boldsymbol{\xi}$ . Having denoted the stress induced by the eigenvector  $\mathbf{A}_\alpha$  on a plane, perpendicular to  $\mathbf{x}_2$ , as  $\mathbf{v}_{0\alpha}$ , we may express the stress induced by the displacement vector  $\boldsymbol{\xi}$  as  $\mathbf{v} = \mathbf{v}_{0\alpha} \xi_\alpha$ . Introducing the matrix  $\mathbf{v}_0$ , element  $v_{0i\alpha}$  of which describes the traction, induced by the displacement eigenvector  $\mathbf{A}_\alpha$  in the direction  $\mathbf{x}_i$ , we express the traction as  $v_i = \xi_\alpha v_{0i\alpha}$ , or  $\mathbf{v} = \mathbf{v}_0 \boldsymbol{\xi}$ . On the other hand, from the expression for  $\mathbf{X}$  it follows that  $\boldsymbol{\xi} = \mathbf{A}^{-1} \mathbf{X}$ . Finally, we are in a position to give an expression for the traction in the most convenient form:

$$\mathbf{v} = \mathbf{Z}_1 \mathbf{X}, \quad (6)$$

where

$$\mathbf{Z}_1 = \mathbf{v}_0 \mathbf{A}^{-1} \quad (7)$$

is an Hermitian matrix called the surface impedance tensor. Ingebrigtsen and Tonnig, intending to stress the fact that the surface impedance tensor isn't defined unambiguously, and the one given in the above expression is only one possibility out of 20, denoted it by  $\mathbf{Z}_1$  instead of  $\mathbf{Z}$ .

The Eq.6 provides a fundamental relation between particle displacement and stress. The relation depends on material properties, the velocity, and choice of the unit vectors  $\mathbf{m}$  and  $\mathbf{n}$  ( $\mathbf{x}_1$  and  $\mathbf{x}_2$ ), but it is independent of the coordinate in the reference frame chosen. The magnitude introduced will be of great importance later, when treating transonic states and addressing directly the problem of existence of the surface acoustic waves.

### Subsonic surface acoustic waves and their existence

Let's choose a subsonic velocity  $v$  and consider three solutions of Eq.3 with positive imaginary part of  $p$ . The reference frame is as previously:  $\mathbf{m} = (1, 0, 0)$  and  $\mathbf{n} = (0, 1, 0)$ . The only difference is that now our body is a half-space having a boundary, perpendicular to  $\mathbf{n}$ , situated at  $x_2 = 0$  (Fig. 4). The amplitudes of the waves described by the solutions vanish when getting deeper into the body. The three solutions of different amplitudes  $\mathbf{A}_\alpha$  and eigenvalues  $p_\alpha$  can be combined into a linear combination with weighting coefficients  $E_\alpha$ :

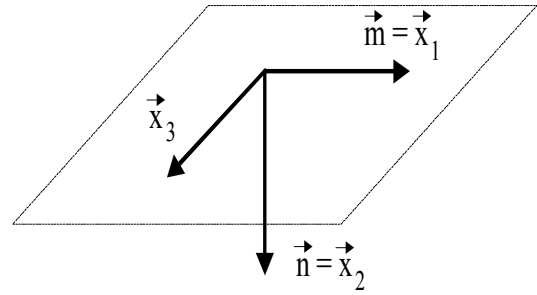


Fig. 4. The geometry of the reference frame in a semiinfinite solid with the boundary, perpendicular to  $\mathbf{n}$ .

$$U_j = E_\alpha u_{\alpha j} = E_\alpha A_{\alpha j} e^{i \frac{\omega}{v} (x_1 + p_\alpha x_2 - vt)} \quad (8)$$

The traction induced by the waves on the surface is

$$T_{2j} = c_{2jkl} \frac{\partial U_k}{\partial x_l} = \quad (9)$$

$$i \frac{\omega}{v} E_\alpha A_{\alpha k} (c_{2jkl} + c_{2jk2} p_\alpha) e^{i \frac{\omega}{v} (x_1 - vt)}$$

As seen from the above formula, the plane  $x_2 = 0$  is left free of traction if

$$E_\alpha A_{\alpha k} (c_{2jkl} + c_{2jk2} p_\alpha) = 0, \quad j = 1 \dots 3. \quad (10)$$

This expression is a set of three algebraic equations with respect to  $E_\alpha$ , having a nontrivial solution if the appropriate determinant is zero. Therefore, if it is possible to find such a speed  $v_R < v_L$  at which the determinant of the matrix  $C_{j\alpha} = A_{\alpha k} (c_{2jkl} + c_{2jk2} p_\alpha)$  (no summation on  $\alpha$  and  $j$ ) vanishes, the SAW may propagate at this subsonic Rayleigh speed. The problem may equivalently be treated employing the surface impedance tensor of Ingebrigtsen and Tonnig, introduced by us earlier. If, for a given orientation and given velocity, there exists a particle displacement vector  $\mathbf{X}$ , such that

$$\mathbf{Z}_1 \mathbf{X} = 0, \quad (11)$$

it means that a wave evoking such a displacement on a plane, perpendicular to  $\mathbf{n}$  ( $\mathbf{x}_2$ ), induces no traction on it. Therefore, such a wave could propagate in a semiinfinite crystal with a boundary, perpendicular to  $\mathbf{n}$  ( $\mathbf{x}_2$ ). Thus the problem of finding the surface wave velocity may be considered as the problem of finding the velocity at which  $\det(\mathbf{Z}_1) = 0$ .

For years it had been an open issue if for any given orientation of a solid of arbitrary anisotropy a solution of a subsonic Rayleigh wave may be found. When considering the problem in 1956, the Irish mathematical physicist J.L.Syngé noticed the following: as the determinant is generally a complex magnitude, its real and imaginary parts are unlikely to vanish at the same velocity simultaneously [3]. Therefore, orientations with no subsonic Rayleigh wave should be expected rather as a rule than an exception. Nevertheless, attempts to find such a forbidden direction failed at that time. Stroh in 1962 showed that the determinant to be considered could be replaced by the determinant of some symmetric purely real matrix. Thus, Syngé's analysis was not as restrictive as originally believed. Unfortunately, Stroh's untimely death stopped his effort in this field. A formalism of the surface impedance tensor by Ingebrigtsen and Tonnig, who were

unaware of Stroh's work, was strikingly similar to that of Stroh [3].

Barnett with coauthors in 1973 showed that whenever a subsonic Rayleigh wave exists, it is unique (there exists no more than one value of velocity  $v_R < v_L$  at which the free boundary condition may be satisfied). Barnett and Lothe in 1974 [6] revealed that in the case of nonexistence of subsonic Rayleigh waves always an exceptional limiting wave (in other words, a surface-skimming bulk wave, SSBW) exists. The statement of existence made by Chadwick and Smith in 1977 was detailed by Barnett and Lothe in 1985 [1], using the surface impedance tensor  $\mathbf{Z}$  of Ingebrigtsen and Tonning. Their existence theorem can be stated in its general form [3]:

“For a given surface wave geometry  $\mathbf{m}$ ,  $\mathbf{n}$ , where  $\mathbf{m}$  is the propagation direction,  $\mathbf{n}$  is the normal to the half-space boundary, a subsonic surface wave exists unconditionally if the first transonic state is normal. If the first transonic state is exceptional and, therefore, necessarily of type 1, a SSBW exists - this follows from the definition of an exceptional transonic state. In this case a subsonic surface wave may or may not exist; in the case of nonexistence a two-component surface wave propagating at the limiting speed (a so called transonic surface wave) may exist.”

The statement in a more rigorous form based upon the properties of the surface impedance tensor was obtained by Barnett and Lothe and given in [1]. The authors considered the eigenvalues of the tensor, i.e., the problem which may be regarded as finding such  $\lambda$  that a vector  $\mathbf{U}$ , satisfying

$$\mathbf{Z}_1 \mathbf{U} = \lambda \mathbf{U}, \quad (12)$$

exists. A value of  $\lambda$  is an eigenvalue of  $\mathbf{Z}_1$  if  $\det(\mathbf{Z}_1 - \lambda \delta_{ij}) = 0$ . As it is a third power algebraic equation with respect to  $\lambda$ , it is clear that the tensor has three eigenvalues, some of which may coincide. As  $\mathbf{Z}_1$  is a Hermitian tensor, its eigenvalues are all real.

The authors showed that all the three eigenvalues are positive at  $v=0$  and decrease monotonically with increasing velocity when  $0 < v \leq v_L$ . Moreover, all the three eigenvalues of  $\mathbf{Z}_1$  cannot be strictly positive in  $[0; v_L]$ . Thus at least one eigenvalue either touches the zero axis at  $v_L$  or intersects with the zero axis at  $v < v_L$  and goes below zero further. On the other hand, at most one eigenvalue may be negative when  $v < v_L$ . As mentioned previously, if  $\det(\mathbf{Z}_1(v)) = 0$ , a solution satisfying the free surface condition exists at this velocity. However, if  $\det(\mathbf{Z}_1(v)) = 0$ , at least one of its eigenvalues vanishes. Therefore, such a solution at the velocity chosen exists, if an eigenvalue of  $\mathbf{Z}_1$  is zero. Thus from the fact that all the eigenvalues cannot be strictly positive it follows directly that in  $[0; v_L]$  there necessarily exists such a value of velocity at which a solution satisfying the free surface condition is possible. From what is said follows that the necessary and sufficient condition for the existence of the subsonic surface wave is that one of the eigenvalues of  $\mathbf{Z}_1$  be negative at the limiting velocity  $v_L$ . Such a wave is unique since the only one eigenvalue that may become negative in  $[0; v_L]$  intersects with the  $\lambda=0$  axis once, as  $\partial \lambda_i / \partial v < 0$ . In the case of nonexistence of the subsonic surface wave at least one eigenvalue of  $\mathbf{Z}_1$  is zero at  $v_L$ . Generally, a zero eigenvalue at  $v_L$  may be caused by

the possibility of existence of a two-component surface wave solution as well as an exceptional limiting wave. Two zero eigenvalues at once may be caused by the existence possibility of the both.

## Conclusions

The paper by Barnett and Lothe [1] provides a means for determining whether a subsonic surface (Rayleigh) wave could propagate on a given cut of a given anisotropic solid in a given direction. This can be done by calculating Ingebrigtsen and Tonning's surface impedance tensor Eq.7 and its eigenvalues at the limiting velocity of the first transonic state. If one of them is negative, a subsonic surface wave with  $v_R < v_L$  exists. If there is no negative eigenvalue, no subsonic surface wave exists. The zero eigenvalues of the tensor (at least one such eigenvalue is always present in the latter case) may be caused by a surface-skimming bulk wave or a two-component surface wave.

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## Paviršinių akustinių bangų sklidimas anizotropiniuose kūnuose Reziumė

Apžvelgiami darbai, kuriuose nagrinėjama paviršinių akustinių Relėjaus bangų sklidimo anizotropiniuose kietuosiuose kūnuose problema. Pateikiamas tuose darbuose naudojamo Ingebrigtseno ir Tonningo paviršiaus impedanso tenzorius išvedimas. Aptariamas Barnetto ir Lothes pasiūlytas metodas, leidžiantis nustatyti, ar pasirinktos orientacijos kietajame kūne gali sklirti paviršinės akustinės bangos.

Pateikta spaudai: 2000 12 12

DOI: 10.5755/j01.u.37.4.8001