# Application of acoustic method for determination of coordinates of leakage in cavities bounded by large surfaces

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#### Introduction

An acoustic correlation measuring method is successfully applied for determination of co-ordinates of leakage in the pipelines transporting gas products [1]. When performing mathematical calculations the spatial dimensions of the pipelines are not estimated [2,3]. In that case it is assumed that the pipeline is a one-dimensional formation. But very often the leakproofness of various reservoirs, vessels and other hollow spaces bounded by large dimension surfaces are investigated. When exploring such objects two-dimensional coordinates are needed for determination of the place of leakage. In that case, to determine the location of leakage points on the surface of cavities the two- dimensional problem has to be solved.

#### **Theoretical analysis**

Let us assume that there is a conventional cavity wall surface. The gas gushing at the leakage points of that surface generates acoustical noises. These noises propagating on the surface of that plane reach the points  $F_0$ and  $F_1$ . At these points electroacoustical transducers register acoustical noises. If the speed of sound in the wall of cavity is c, and the time of propagation of acoustic noises from the leakage place S to the focal point  $F_0$  is  $t_0$ , then the distance  $r_0$  from the leakage place S to the focus point  $F_0$  may be determined by equation  $r_0=ct_0$ . By analogy, the distance  $r_1=ct_1$ , where  $t_1$  is the time of propagation of noises from S to the focus point  $F_1$ . The difference  $\Delta t_1$  of times of propagation of acoustical noises from the leakage point S to the electroacoustical transducers  $F_0$  and  $F_1$  may be determined as

$$\Delta t_1 = t_0 - t_1 = \frac{r_0 - r_1}{c} \,. \tag{1}$$

After applying the theorem of cosines it may be written

$$r_1^2 = r_0^2 + d_1^2 - 2r_0 d_1 \cos(\gamma_1).$$
<sup>(2)</sup>

Here  $d_1$  is the distance between electroacoustical transducers,  $\gamma_1$  is the angle between  $d_1$  and  $r_0$ . Solving the system of Eq. 1 and 2 enables to eliminate  $r_1$  and to determine  $r_0$ :

$$r_0 = \frac{c^2 (\Delta t_1)^2 - d_1^2}{2[c\Delta t_1 - d_1 \cos(\gamma_1)]}.$$
(3)

How one can see from Eq 3 all terms on the right side, except the angle  $\gamma_1$ , are measurable or selectable parameters. Thus, the distance  $r_0$  is the function of angle  $\gamma_1: r_0=r_0(\gamma_1)$ .



Fig.1. Determination of leakage coordinates on the plane by using one pair of electroacoustical transducers

Since the difference of distances of propagation of acoustical noises from the leakage place *S* to the electroacoustical transducers  $F_0$  and  $F_1 \ \Delta r = r_0 - r_1$  is constant, the radius vector will draw a hyperbola on the plane when the angle  $\gamma$  is changed (Fig.1). It means that any point of hyperbola may be the point of leakage. With the purpose to avoid ambiguity, we supplement the measuring system with one more electroacoustical transducer located at the point  $F_2$  (Fig.2). When analysing the propagation of acoustical noises from the leakage place *S* to electroacoustical transducers  $F_0$  and  $F_2$ , it will be obtained that

$$r_0 = \frac{c^2 (\Delta t_2)^2 - d_2^2}{2[c\Delta t_2 - d_2 \cos(\gamma_2)]}.$$
(4)

It can be seen from Fig.2 that  $\gamma_2 = \gamma_1 - \alpha$ , where  $\alpha$  is an angle between the straight lines  $F_0F_1$  and  $F_0F_2$ . When



Fig.2. Determination of leakage co-ordinates by using two pairs of electroacoustical transducers



Fig.3. Probable points of leakage: a - 2, b - 4

equalizing Eq. 3 and 4 and replacing the angle  $\gamma_2$  with the expression  $\gamma_2 = \gamma_1 - \alpha$ , one can obtain

$$\frac{c^2 \Delta t_1^2 - d_1^2}{c \Delta t_1 - d_1 \cos(\gamma_1)} = \frac{c^2 \Delta t_2^2 - d_2^2}{c \Delta t_2 - d_2 \cos(\gamma_1 - \alpha)} .$$
(5)

After the transformation of this equation  $2^{2}$ 

$$(c^{2}\Delta t_{1}^{2} - d_{1}^{2})[c\Delta t_{2} - d_{2}\cos(\gamma_{1})\cos(\alpha) - d_{2}\sin(\gamma_{1})\sin(\alpha)] = (c^{2}\Delta t_{2}^{2} - d_{2}^{2})[c\Delta t_{1} - (6)$$

 $d_1 \cos(\gamma_1)$ ]

it ca

and introducing the notations

$$A = (c^{2} \Delta t_{1}^{2} - d_{1}^{2})c\Delta t_{2},$$
  

$$B = (c^{2} \Delta t_{1}^{2} - d_{1}^{2})d_{2}\cos(\alpha),$$
  

$$C = (c^{2} \Delta t_{1}^{2} - d_{1}^{2})d_{2}\sin(\alpha),$$
  

$$D = (c^{2} \Delta t_{2}^{2} - d_{2}^{2})c\Delta t_{1},$$
  

$$E = (c^{2} \Delta t_{2}^{2} - d_{2}^{2})d_{1},$$
  
n be obtained:  
(7)

$$A - B\cos(\gamma_1) - C\sin(\gamma_1) = D - E\cos(\gamma_1).$$
 (8)

In Eq. 8 we express  $\sin(\gamma_1)$  by  $\cos(\gamma_1)$  and introduce notations:

$$M = (B - E)^{2} + C^{2}$$
  

$$N = (A - D)(B - E).$$
  

$$L = (A - D)^{2} - C^{2}$$
(9)

After some algebraic operations one can obtain a quadratic equation:

$$M\cos^{2}(\gamma_{1}) - 2N\cos(\gamma_{1}) + L = 0.$$
 (10)

Solving this equation one can find out an expression for the angle  $\gamma_1$ 

$$\gamma_1 = \arccos\left(\frac{N \pm \sqrt{N^2 - ML}}{M}\right). \tag{11}$$

For simplification of calculations when using Eq. 11 one can return to the notations applied in Eq. 7:

$$\gamma_{1} = \arccos\{[(A-D)(B-E)\pm C\sqrt{(B-E)^{2} - (A-D)^{2} + C^{2}}]/[(B-E)^{2} + C^{2}]$$
(12)

After analysing Eq. 12 it can be seen that in the interval  $0 \le \gamma_1 < 2\pi$  depending on the coordinates of the

leakage and the location of electroacoustical transducers the Eq. 12 may have 2, 3 or 4 real solutions (Fig.3).

With the purpose to determine which of the determined values of the angle  $\gamma_1$  between  $d_1$  and  $r_0$  is true, let us introduce one more electroacoustical transducer  $F_3$ . By analogy to the case described earlier, when coupling the new transducer  $F_3$  with the transducer  $F_0$ , one can obtain

$$r_0 = \frac{c^2 (\Delta t_3)^2 - d_3^2}{2[c\Delta t_3 - d_3 \cos(\gamma_1 - \beta)]}.$$
 (13)

Here  $\beta$  is the angle between the straight lines connecting the pairs of transducers  $F_0F_1$  and  $F_0F_3$ . Substituting the value  $\gamma_1$  in Eq. 13 with the solution, obtained by solving Eq. 12, the values of  $r_0$  can be calculated. After that we compare the results of calculation obtained by using the algorithms 3 and 13 when the values of angle  $\gamma_1$  are the same. The value of  $r_0$ , which is the same when applying Eq. 3 and 13, is considered to be the true distance from the electroacoustical transducer  $F_0$ . The value of angle  $\gamma_1$ , which corresponds this distance, is the true value of angle between  $d_1$  and  $r_0$  (Fig.4).



Fig.4. Unambiguous determination of leakage coordinates by using three pairs of independent electroacoustical transducers

Analysis of the results of calculation shows that when using three independent pairs of electroacoustical transducers and applying algorithm (3) three hyperbolas are obtained and they have only one common point of

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intersection. This point corresponds to the leakage place on the plane of measurements. In this way it is possible to determine the position of leakage on the plane unambiguously when the values of the angle  $\gamma_1$  and the distance  $r_0$  are known.

### Conclusion

The great actuality of assurance of the leakproofness of reservoirs, vessels and other cavities bounded by the large dimension surfaces was shown.

Noval method and algorithms were proposed for determination of coordinates of the noise source in the two-dimensional area.

The proposed method and algorithms might be used while developing ultrasonic systems for detection of leakage and determination of its coordinates when the cavities (hollows) are bounded by large plane surfaces.

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# Akustinio metodo taikymas dideliais paviršiais apribotų ertmių nesandarumų koordinatėms nustatyti

Reziume

Nustatant dideliais paviršiais apribotų talpyklų nesandarių vietų koordinates, būtina spręsti dvimatį uždavinį. Tokiu atveju, koreliaciniu metodu nustatant galimo nehermetiškumo koordinates plokštumoje pagal akustinių triukšmų sklidimo nuo nuotėkio vietos iki elektroakustinių keitiklių laikų skirtumą, rezultatas gaunamas daugiareikšmis. Galimos nesandarios vietos atitinka hiperboles, kurių židiniuose išdėstyti elektroakustiniai keitikliai. Naudojant trečią elektroakustinį keitiklį, nesandari vieta plokštumoje gali būti aprašoma lygčių sistema, kuri priklausomai nuo hermetiškumo pažeidimo vietos ir elektroakustinių keitiklių tarpusavio padėties gali turėti 2, 3 arba 4 realius sprendinius. Parodoma, kad, siekiant vienareikšmiško sprendinio, būtinas dar vienas elektroakustinis keitiklis. Pasiūlyta metodika ir algoritmai gali būti taikomi kuriant ultragarsines sistemas didelių talpyklų ir rezervuarų nesandarumams aptikti ir jų koordinatėms nustatyti.

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