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Theoretical background for the supply and dosing mechanisms under elastic vibrations

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Introduction

There can be developed many various mechanisms and devices, which employ vibrations for the transportation and dosing of fluids and particle type materials. For this purpose piezoactive materials have been used. Their physical and technical parameters are especially suitable for development of precision devices [1,2].

A few schemes of the mentioned mechanisms will be analyzed. The main vibration characteristics of the elements, which influence the dynamics of transferred bodies, are evaluated using the method of finite elements [3,4,5].

Supply of fluids caused by vibrating rigid body under deformation

Depending on the design, transportation elements can be of the open (Fig. 1) and closed (Fig. 1) type.

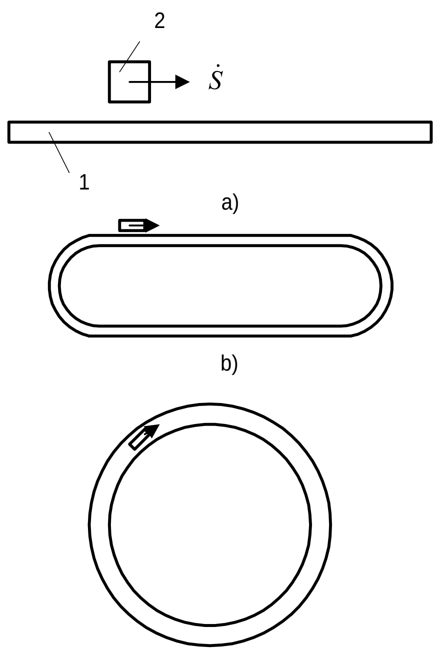

Fig.1 **Models of supply and dosing devices: a - open type, b - closed type**

Transported objects, namely particles of a solid and the fluid, are also shown in the figure above. Finite element method is applied for the investigation of dynamics of transporting elements. The operation principle of the devices in hand is based upon the appearance of friction forces between the transporting elements and transported objects. The friction forces are caused because of the normal reaction in the pair element-object. Usually a dry friction coefficient is used in these calculations, and the components of viscous friction are used more rarely. The friction force in an element acting upon some object and the external forces cause the motion of bodies or fluids.

Various types of vibrators can be used for excitation of vibrations in the transporting element. In precise equipment ceramic materials are most often used as vibration sources. The traveling wave deformations are the most suitable to be excited using a few vibration sources of the same frequency, which are fixed to the transporting element at a specified distance and choosing appropriate vibration phases.

There are investigated open- and closed-type transporting elements, the surface vibrations of which carry viscous fluids. Dynamics of the structural elements is described by the following differential matrix equations:

$$[M]\{\ddot{\delta}\}+[c]\{\delta\}+[K]\{\delta\}+\{F\}=0\,, \tag{1}$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, {F} is the load vector, {δ} is the vector of generalized displacements.

The problem of eigen values is solved by the generalized Jacobi method [6]. The finishing condition for iteration process is the relative error of eigen values in sequential iterations.

The calculations of the dynamic equation are performed and the obtained results give a practical base for the improvement of existing transportation mechanisms and development of new designs.

The dynamical equation, when there is no damping and excitation, becomes of the following form:

$$[K]\{\delta\}+[M]\{\ddot{\delta}\}=0\,. \tag{2}$$

It has a solution

$$\{\delta\}=\{\delta\}\cos\omega t, \tag{3}$$

when

$$([K]-\omega^2[M])\{\delta_0\}=0. \tag{4}$$

This is possible only if

$$\left|[K]-\omega^2[M]\right|=0. \tag{5}$$

The frequencies and shapes $\{\delta_0\}$ are obtained in the above described way. Own forms are orthogonal and are normalized as follows

$$\{\delta_0\}^T[M]\{\delta_0\}_j=\delta_{ij}\,, \tag{6}$$

where (δ_{ij}) is the Kroneker symbol.

A model of a thin bar filled with fluid

The fluid is described by the following system of equations:

$$\begin{cases}\rho\left(\dfrac{\partial\bar{u}}{\partial t}+\bar{u}\dfrac{\partial\bar{u}}{\partial x}\right)=-\dfrac{\partial p}{\partial x}-h\left|\bar{u}-\dfrac{\partial u}{\partial t}\right|\left(\bar{u}-\dfrac{\partial u}{\partial t}\right)\\ \rho\dfrac{\partial\bar{u}}{\partial x}+c^{-2}\dfrac{\partial p}{\partial t}=0,\end{cases}\tag{7}$$

where $\bar{u}$, u are the longitudinal velocities of the fluid and the displacement of the bar. The friction force is proportional to the square of relative velocity, $c^{-2}=\rho/\bar{E}$, where $\bar{E}$ is the fluid stiffness modulus, although for evaluating the stiffness of walls another value of $\bar{E}$ can be used.

$$\begin{aligned}&\rho\left(\frac{\partial^2 u}{\partial t^2}+\frac{\partial\bar{u}}{\partial t}\frac{\partial\bar{u}}{\partial x}+\bar{u}\frac{\partial^2\bar{u}}{\partial x\partial t}\right)\frac{\partial}{\partial x}\left(E\frac{\partial\bar{u}}{\partial x}\right)+\\&+2h\left|\bar{u}-\frac{\partial u}{\partial t}\right|\left(\frac{\partial\bar{u}}{\partial t}-\frac{\partial^2 u}{\partial t^2}\right)=0.\end{aligned}\tag{8}$$

Fluid pressure causes a longitudinal force in the bar:

$$\frac{\partial p}{\partial t}=-\bar{E}\frac{\partial\bar{u}}{\partial x}.\tag{9}$$

There is investigated an element interpolated by a first order Ermitt polynomials. In order to select the derivatives at the nodes $\left(\dfrac{dx}{d\xi}\right)i$ and $\left(\dfrac{dy}{d\xi}\right)i$ Lagrange square functions there are used:

$$\begin{aligned}&\left(\frac{dx}{d\xi}\right)_1=\frac{-3x_1+4x_{12}-x_2}{2},\\&\left(\frac{dx}{d\xi}\right)_2=\frac{x_1-4x_{12}-3x_2}{2},\end{aligned}\tag{10}$$

where x_{12} is the coordinate of an intermediate node determining the geometry. As node parameters are bar displacements and longitudinal fluid velocity and their derivatives with respect to S. The fluid velocity is interpolated as a scalar quantity. According to common displacements and value of the velocity it is calculated on the past moment:

$$\begin{aligned}&A=\frac{\partial v}{\partial s}=[N_v']\{\sigma\},\\&M=EF\frac{\partial u}{\partial s}=EF[N_u']\{\sigma\},\\&P=-\overline{EF}\frac{\partial\bar{u}}{\partial s}=-\overline{EF}[N_u']\{\sigma\},\\&\bar{\bar{v}}=[N_{\bar{u}}']\{\delta\},\\&\bar{\bar{\alpha}}=[N_{\bar{u}}]\{\dot{\delta}\},\\&\bar{\bar{h}}=hF'\left|\bar{u}-\frac{\partial u}{\partial t}\right|=h\bar{F}\left|[N_{\bar{u}}]\{\delta\}-[N_u]\{\dot{\delta}\}\right|,\\&p=p_{ij}+\dot{p}\Delta t,\end{aligned}$$

were $\bar{F}$ is the fluid cross-section area; Δt is the step; p_{ij} is p at the past moment in the particular integration point, later given p_{ij} -is p.

Matrix expressions have the following shape:

$$\begin{aligned}&[K]=\int\Big(\Big(-\bar{\bar{v}}^2\bar{m}+EFA^2/2+M/2+p\Big)[N_v']^T[N_v']+\\&+EFA/2\Big([N_u']^T[N_v']+[N_v']^T[N_u']\Big)+\Big(EF-\bar{\bar{v}}^2\bar{m}\Big)[N_u']^T[N_u']+\\&+E[N_v'']^T[N_v'']+\beta_u[N_u]^T[N_u]+\beta_v[N_v]^T[N_v]+\\&+\bar{\bar{\alpha}}\bar{m}\Big([N_u]^T[N_u']+\beta_v[N_v]^T[N_v']\Big)-\bar{\bar{h}}[N_u]^T[N_{\bar{u}}]+\\&+\bar{m}\bar{\bar{a}}[N_{\bar{u}}]^T[N_{\bar{u}}]++E\bar{F}[N_{\bar{u}}']^T[N_{\bar{u}}']\Big)ds,\\&[c]=\int\Big(\Big(\alpha_u+\bar{\bar{h}}\Big)[N_u]^T[N_u]+\alpha_v[N_v]^T[N_v]+\\&+2\bar{m}\bar{\bar{v}}\Big([N_u]^T[N_u']+[N_v]^T[N_v']\Big)+\\&+\bar{m}\bar{\bar{v}}[N_{\bar{u}}]^T[N_{\bar{u}}']+2\bar{\bar{h}}[N_{\bar{u}}]^T[N_{\bar{u}}']\Big)ds,\\&[M]=\int\Big((m+\bar{m})\Big([N_u]^T[N_u]+[N_v]^T[N_v]\Big)+\\&+\bar{m}[N_{\bar{u}}]^T[N_{\bar{u}}]-2\bar{\bar{h}}[N_{\bar{u}}]^T[N_u]\Big)ds.\end{aligned}\tag{11}$$

where β_u, β_v – longitudinal and traverse elastic Winkler base stiffnesses; $\bar{m}=p\bar{F}$, α_u, α_v – longitudinal and traverse damping of the bar.

Common model and calculation of interaction between fluid and elastic body

The motion of fluid can be described by the following system of equations:

$$\begin{aligned}&\rho c^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\frac{dp}{dt}=0,\\&\rho\frac{du}{dt}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\frac{4}{3}\mu\frac{du}{dx}\right)-\frac{\partial}{\partial x}\left(\frac{2}{3}\mu\left(\frac{dv}{dy}+\frac{\partial w}{\partial z}\right)\right)+\\&+\frac{\partial}{\partial y}\left(\mu\left(\frac{du}{dy}+\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial x}\left(\mu\left(\frac{du}{dyz}+\frac{\partial w}{\partial x}\right)\right),\ldots\end{aligned}\tag{12}$$

After some substitutions:

$$\frac{\partial}{\partial x}\left(\rho c^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right)+\frac{d}{dt}\left(\rho\frac{du}{dt}-\bar{\bar{\mu}}u\right)=0.\tag{13}$$

Taking into account some convective members:

$$\begin{aligned}&\frac{du}{dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z};\ldots\\&\frac{d^2u}{dt^2}=\frac{\partial^2u}{\partial t^2}+\frac{\partial u}{\partial\quad t}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial t}\frac{\partial u}{\partial y}+\frac{\partial w}{\partial t}\frac{\partial u}{\partial z}+\\&+2\left(u\frac{\partial^2u}{\partial x\partial t}+v\frac{\partial^2u}{\partial y\partial t}+w\frac{\partial^2u}{\partial z\partial t};\ldots\right).\end{aligned}\tag{14}$$

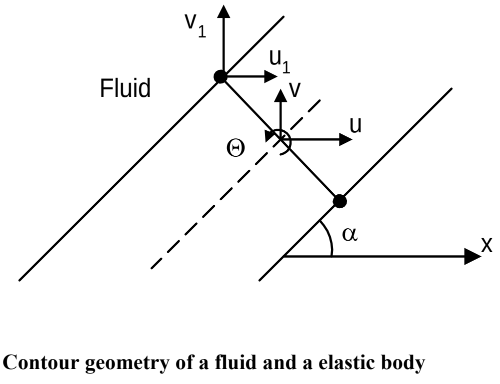

Fig. 2. **Contour geometry of a fluid and a elastic body**

The convective members when μ =0, are estimated as loading. The axially symmetric case is investigated and the matrix equations of elastic body and fluid can be derived, then:

$$[M]=\int[N]^T\rho[N]dv;$$

$$[C]=\int\left([N]^T\bar{\mu}[N]+[B]^T[D][B]\right)dv;$$

$$[K]=\int\left([B]^T[D][B]+[B]^T\rho C^2[\bar{B}]\right)dv;$$

Here: $[\bar{D}]=[\tilde{D}]+\beta[D]$,

β is the coefficient of internal damping of elastic body;

ρ is the density of elastic body or density of fluid;

μ is the external damping of elastic body;

$[\bar{D}]$ is the relation matrix between stresses and deformations in fluid;

$[D]$ is the matrix of elastic constants of elastic body;

C is the sound velocity in fluid;

$[B]$ is the relation matrix of volume extension with displacement.

Using the method of Lagranze--Euler the convection can be evaluated. Initial $\{\delta\}, \{\dot{\delta}\}, \{\ddot{\delta}\}$ values can be recalculated for internal nodes, taking into account their displacement with constants u_i, $\dot{u}_i, \ddot{u}_i$, the pitch, adjusting interpolation.

$a_1+a_2+a_3y+a_4xy+a_5(x^2+y^2).$

The construction (Fig.3) of natural (eigen) design is investigated. Here the radial coordinate of the first node – 4; of the 21th - 8; 3 node – 3,8; the first nodes vertical coordinate 0; 5 node – 4; 11 node – 0,2; 15 node – 4,2. The coordinates of nodes are described , using Sirendin's square rectangular. It is accepted that ρ = 0,002; c^2=80. The velocity of all surface is assumed zero in two directions (because of viscous fluid). The values of natural vibrations are obtained: 4,739; 4,925; 4,962; 7,545; 7,832; 8,703; 11,35; 11,91.

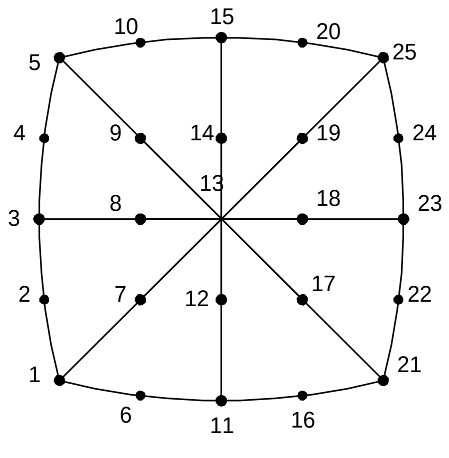

Fig. 3. **Fluid discreditation**

The construction was investigated, the zero boundary conditions are assumed for vertical walls, and for horizontal walls – periodical. Two rows of right nodes are of elastic body. The natural forms are calculated, when E=0.8·10^4; ν=0,3. The parameters of fluid are ρ=0,001; c^2=1440000. The radial coordinate of the first node 2; 4th node – 8; vertical coordinate of the first node – 0; 7th node - 12. The value of the first natural vibrations: 0; 0; 0; 318,0; 322,6; 495,3; 497,0; 632,4; 632,5; 706,3; 715,0; 936,3. The first three natural forms in fluid has only vertical displacement.

The dynamics calculation has been done when the parameters of elastic body – β=0,4·10^{-7}; fluid – μ=0,00008. The frequency corresponding to the exciting period 632. General coordinates with loading (Fig.4), when loading is sine curve, amplitude 100, phase F_1 equal 0, and F_2 equal $2\pi/3$, $F_3=4\pi/3$.

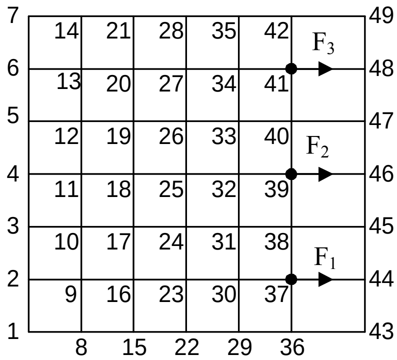

Fig. 4. **Fluid and elastic body discreditation**

Because (as) the system of elastic body is accepting as partial time differential then the value is $\dot{F}_i$. The pitch is equal 1/16 of decreasing period. The initial conditions are zero. The integration is done using Newmark constant average acceleration scheme. After 200 steps the maximum velocity at this moment is in he 27th node of the construction in vertical direction and equal 2,02.

The response of the system to the settled harmonic excitation is calculated. After that the static problem depending on average loading is solved.

The construction (Fig. 4) is analyzed, when F_2=F_3=0, F_1=100 $e^{j\omega t}$. The constant value of velocity proves that there can be vertical displacement at each of the three rows: fluid as elastic body. Zero value of constant velocity for elastic body is obtained too. The calculation results are presented in Table 1. We can see, that reaction to harmonic excitation is displayed only to the velocity in vertical directions, to each row of vertical nodes and distinct vibrations in fluid disappeared.

Table 1 **Calculated rezults**

Nr. of nodes row to the center	1	2	3	4	5	6
Amplitude	8,53	8,53	19,9	58,6	189	159
Phase, rad	$-3\cdot10^{-5}$	3,1415	$-3\cdot10^{-5}$	3,1415	$-3\cdot10^{-5}$	$-2\cdot10^{-5}$

In case of torsion we can assume, that displacement according to the angular coordinate Θ(Fig.2) in the cylindrical system of coordinates is function $W(r,z)$, then:

$$[B]=\begin{bmatrix} \dfrac{\partial N_1}{\partial r}-\dfrac{N_1}{r} & \ldots \\ \dfrac{\partial N_1}{\partial z} & \ldots \end{bmatrix}. \tag{15}$$

In this problem the compression of fluid is insignificant because of volume pressure which is 0.

The response to harmonic excitation of the construction which height is 12, r is from 0 to 6; 7 – the rows of vertical and horizontal nodes, is calculated. When r=0 and r=6 – we have kinetic excitation; two rows are external elements of elastic body; others are of fluid; $E=0{,}8\cdot10^4$; $\nu=0{,}3$; for the elastic body $\rho=0{,}002$; $\beta=0{,}4\cdot10^{-7}$; $\mu=0{,}00008$; for fluid $\rho=0{,}001$, frequency 632; the amplitude of kinetics excitation 100.

The calculated results are presented in Table 1. The obtained results make it possible to conclude that the fluid motion is not depending on z and the amplitude is decreasing from the wall's excitation to the center direction.

Conclusions

For investigation of dynamics of the fluid supply mechanism elements the generalized mathematical models were constructed and the theoretical basis for their operations were developed. The theoretical investigation using analytical and numerical methods and applying integration procedure of differential equations with respect to the vibration eigen values was carried out. 3-D model of a bar-type element, which evaluates the displacement of convective components, inertia and deplanation of spin-type element was developed.

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Tampriai virpančių tiekimo ir dozavimo mechanizmų teoriniai pagrindai

Reziumė

Straipsnyje pateikiami tampriai virpančių skysčių tiekimo ir dozavimo mechanizmų teoriniai pagrindai. Tiriami atviri ir uždari transportavimo elementai, kurių paviršiaus virpesiai neša klampius skysčius. Pateikiamas plono strypo su skysčiu modelis, bendras modelis ir skysčio sąveikos su tampriu kūnu skaičiavimas ir rezultatai.

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