ISSN 1392-2114 ULTRAGARSAS, Nr.3(44). 2002.

Hybrid numerical – experimental holographic fluid interferometry: two-dimensional compressible fluid films

M. Ragulskis[1], A. Palevičius[1], L. Ragulskis[2]

[1]*Department of Mathematical Sciences, Kaunas University of Technology*

[2]*International Studies Center, Kaunas University of Technology*

[3]*Vytautas Magnus University*

Introduction

Holography is a powerful flow diagnostic for visualising and analyzing a variety of physical and engineering problems. Fluid holography in its turn enables effective analysis of high speed flow problems, high frequency vibrations of micro-scale components and fluids in dosing and contacting units, high speed processed taking place in biological and chemical microsystems. The investigation of the high frequency vibrations of the fluid is an important problem in the design of various devices.

Though the production stage of the interferograms is technically not extremely complicated, the interpretation of the produced fringes faces a huge number of mathematical and numerical problems. That is firstly related with the complex geometry of the phase-shifting media. Under such circumstances the density can change along the line of the sight, and the density is no longer proportional to phase. The reconstruction of the phase involves application of an Abel deconvolution [4] what makes the direct fringe interpretation almost impossible. Therefore, development of hybrid numerical – experimental fluid holographic methods is important both for the interpretation of experimental results, both for the analysis of systems in the virtual environments by generating realistic interferograms. In this paper the method of holographic interferometry is used for the analysis of the two-dimensional fluid problem.

Numerical model of the system

The two-dimensional problem of vibrations of the potential ideal compressible fluid under the assumption of constant density in the status of equilibrium is described by the equation [1, 2]:

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}=0\,, \qquad (1)$$

where ϕ is the velocity potential; c – the velocity of sound; t – time; x and y – orthogonal cartesian coordinates.

The free surface boundary condition on the surface $y = const$ is:

$$\frac{\partial\phi}{\partial y}=-\frac{1}{g}\frac{\partial^2\phi}{\partial t^2}\,, \qquad (2)$$

where g is the acceleration of gravity.

The stiffness and mass matrixes of the fluid are:

$$K=\iint B^T B dxdy\,, \qquad (3)$$

$$M=\iint N^T\frac{1}{c^2}Ndxdy+\int \overline{N}^T\frac{1}{g}\overline{N}dx \qquad (4)$$

where N is the row matrix of the shape functions of the finite element, B is the matrix the rows of which are the derivatives of the shape functions with respect to x and y, $\overline{N}(x)$ is the row matrix of the shape functions of the one-dimensional finite element at the free surface $y = const$.

In order to obtain the nodal velocities the conjugate approximation [3] is used. The values of velocities at the points of numerical integration of the finite elements are obtained as:

$$\begin{Bmatrix}\dfrac{\partial\phi}{\partial x}\\ \dfrac{\partial\phi}{\partial y}\end{Bmatrix}=B\delta\,, \qquad (5)$$

where δ stands for the vector of nodal values of the velocity potential for the analyzed finite element. Then the nodal values of the velocities are obtained by solving the following systems of linear algebraic equations:

$$\left[\sum\left(\iint N^T N dxdy\right)\right]\cdot\left[\{\delta_u\}\quad\{\delta_v\}\right]=\sum\left(\iint N^T\begin{bmatrix}\dfrac{\partial\phi}{\partial x} & \dfrac{\partial\phi}{\partial y}\end{bmatrix}dxdy\right), \qquad (6)$$

where the summation operator stands for the direct stiffness procedure [5, 6], vector columns $\{\delta_u\}$ and $\{\delta_v\}$ are the nodal values of the velocities in the x and y directions respectively of the global structure.

Construction of the holographic image

The phase of the light from the laser beam is [4]:

$$\Psi(x,y)=\frac{2\pi}{\lambda}\left[n_0-n_{flow}(x,y)\right]h\,, \tag{7}$$

where *h* is the distance that the light travels through the fluid, λ is the wavelength of the laser beam, n_0 and n_{flow} are the refractive indexes in the initial and flow conditions respectively.

The refractive index is expressed as [4]:

$$n(x,y)=1+\beta\frac{\rho(x,y)}{\rho_0}\,, \tag{8}$$

where ρ_0 is the density constant in the region of the flow in the status of equilibrium, β is the constant of proportionality.

From the previous relationships it follows that:

$$\Psi(x,y)=\Psi_0-k\rho_{flow}(x,y), \tag{9}$$

where the initial phase ψ_0 and the coefficient of proportionality *k* are expressed like:

$$\Psi_0=\frac{2\pi}{\lambda}\beta h\,, \tag{10}$$

$$k=\frac{2\pi}{\lambda}\frac{\beta}{\rho_0}h\,. \tag{11}$$

Further it is assumed that:

$$\tilde{\rho}(x,y,t)=\rho_{flow}(x,y,t)-\rho_0\,, \tag{12}$$

where the deviation of the density from the density in the status of equilibrium is small:

$$\left|\tilde{\rho}(x,y,t)\right|<<\rho_0\,. \tag{13}$$

Then:

$$\Psi(x,y)=\overline{\Psi_0}-k\tilde{\rho}(x,y,t)\,, \tag{14}$$

where

$$\overline{\Psi_0}=\Psi_0-k\rho_0\,. \tag{15}$$

From [1, 2] it is known that:

$$\frac{\partial\rho}{\partial t}=-\frac{1}{c^2}\rho_0\frac{\partial^2\phi}{\partial t^2}\,. \tag{16}$$

Further it is assumed that the density and the velocity potential are harmonically varying in time:

$$\rho_{flow}(x,y,t)=\rho_0+\tilde{\rho}^*(x,y)\cos(\omega t), \tag{17}$$

and

$$\phi(x,y,t)=\tilde{\phi}^*(x,y)\sin(\omega t), \tag{18}$$

where ω particularly can coincide with the frequency of oscillations of the appropriate eigenmode.

Eq.(16) together with the previous equations gives:

$$\tilde{\rho}^*(x,y)=-\frac{1}{c^2}\rho_0\omega\tilde{\phi}^*(x,y). \tag{19}$$

So after performing the initial calibration of the phase of the laser beam its intensity *I* for the stroboscopic image can be expressed as:

$$I(x,y)=\cos^2\left(a\tilde{\phi}^*(x,y)\right), \tag{20}$$

where the coefficient *a* can be expressed from equations (14) and (19):

$$a=\frac{1}{c^2}\rho_0\omega k\,. \tag{21}$$

Numerical investigation

The rectangular domain is analysed. The upper surface is assumed to be a free surface. The periodic boundary conditions in the *x* direction are assumed: that is the values of the velocity potential on the left and the right boundaries for the same values of the *y* coordinate are assumed to be mutually equal.

The eigenmodes are calculated and on their basis the eigenmodes of nodal velocities are obtained by using the conjugate approximation.

The fifth and the tenth eigenmodes are shown in Fig. 1 and Fig. 2. The obtained stroboscopic holographic images for the same eigenmodes are presented in Fig. 3 and Fig. 4.

Conclusions

The method of holographic interferometry is successfully applied for the two-dimensional problem of vibrations according to the eigenmode by using the stroboscopic method lightening the structure in the state of extreme deflections.

Fig. 1. **The fifth eigenmode (black solid lines) and the mesh in the status of equilibrium (grey lines)**

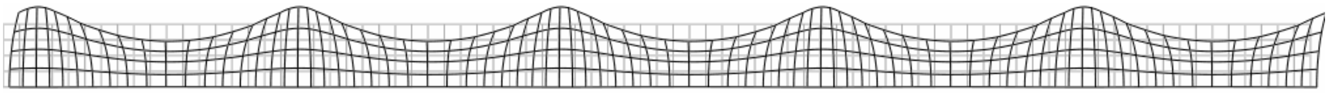

Fig. 2. **The tenth eigenmode (black solid lines) and the mesh in the status of equilibrium (grey lines)**

Fig. 3. **The stroboscopic holographic image for the fifth eigenmode**

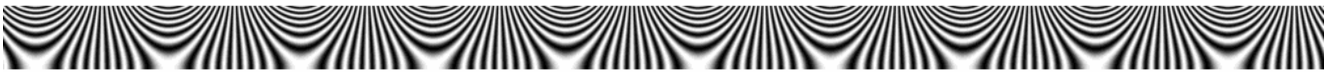

Fig. 4. **The stroboscopic holographic image for the tenth eigenmode**

References

1. **Iershov N. F., Shahvierdi G. G.** The method of finite elements in the problems of hydrodynamics and hydroelasticity. Lieningrad: Sudostroienie. 1984. P. 240.
2. **Brebbia C. A., Tottenham H., Warburton G. B., Wilson J. M.** Vibrations of engineering structures. Berlin, Springer-Verlag. 1985. P. 302.
3. **Segerlind L. J.** Applied finite element analysis. Moscow: Mir. 1979. P. 392.
4. **Houwing A., Takayama K., Koremoto K., Hashimoto T., Mitobe H.** Finite fringe analysis of two dimensional and axially-symmetric flows. Proc. of SPIE. 1983. Vol. 398. P. 174-180.
5. **Zienkiewicz O., Taylor R.** Finite element method: Fluid dynamics. Butterworth Heinemann. London. 2000. Vol. 3. P. 352.
6. **Dhaubhadel M.** Advances in finite element analysis in fluid dynamics. ASME, N.Y., 1994. P. 388.

M. Ragulskis, A. Palevičius, L. Ragulskis

Hibridinė skaitinė - eksperimentinė skysčių holografinė interferografija: dvimatės slegiamos skysčio plėvelės

Reziumė

Sukurtas skaitinis skysčių holografinės interferografijos metodas slegiamo skysčio analizei. Pirmajame etape parodyta, kaip metodas gali būti pritaikomas dvimatėms slegiamo skysčio plėvelėms, virpančioms aukštu dažniu. Metodo plėtros procese išspręsta nemaža skaitinių ir matematinių problemų.

Pateikta spaudai 2002 09 25