Prediction of ultrasonic waveforms in highly attenuating plastic materials

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Introduction

The delay time estimation is one of the most essential procedures in ultrasonic measurements such as thickness measurements, determination of ultrasound velocity, sizing of the defects in non-destructive testing and etc. In simplest cases usually two signals are exploited. Depending on a measurement technique used one of them is the excitation signal or signal reflected by the front boundary of the object, the second one is the signal transmitted through the object under investigation or reflected by the back wall of an object. The time interval between these two signals is the physical quantity necessary to measure. There are many methods used for estimation of this time interval, but the most reliable and accurate are methods based on calculation of a cross-correlation function between these two signals. The signal reflected by the front surface of the object usually is used as a reference signal in a cross-correlation analysis. Such an approach may be imagined as a search in the time domain of another signal, which is very similar to the reference signal. The instant of the best correlation in the time domain corresponds to the instance of the arrival time of the second signal. The main problem, which is met in implementation of this approach, is that the accuracy of this technique depends on a degree of similarity of these two signals. The waveform of the signal transmitted through the object usually is distorted due to different factors, such as attenuation, dispersion, diffraction, geometry of boundaries and etc. Selecting optimal parameters of ultrasonic transducers and measurement distance enable to reduce the influence of some of them, however the distortions caused by attenuation of ultrasonic waves always exist.

In many industrial applications pulse echo measurements are widely used because they enable to perform measurements when only one side access to the object under investigation is available. In this case signals reflected by internal interfaces or non-uniformities in the object may be exploited for measurements and, consequently, for characterization of the internal structure of the object.

In this case for enhancement of the accuracy of measurements it is necessary to predict the waveforms of the ultrasonic signals reflected by interfaces, which may be located anywhere inside the object.

The main objective of the presented research is to develop a technique, suitable for prediction of waveforms of ultrasonic signals reflected by planar interfaces in various materials possessing a high frequency dependent attenuation and dispersion of ultrasonic waves.

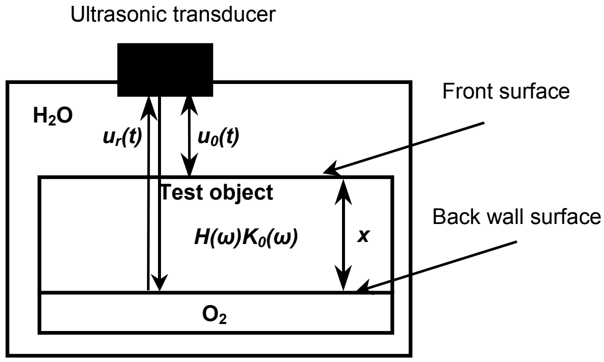

Fig.1. Principle of pulse echo immersion measurements

Prediction approach

Pulse echo measurements usually exploit multiple reflections of ultrasonic waves between internal and external interfaces of the object under investigation. In a simplest case when the object consists of a single planar layer immersed into water for measurements the signals, reflected by the front and back walls, are exploited (Fig.1). The signal $u_r(t)$ reflected by a back surface is distorted in comparison to the reference signal $u_0(t)$ mainly due to a frequency dependent attenuation and velocity dispersion of ultrasonic waves. The waveform of this signal may be predicted from the known reference signal $u_0(t)$ (Fig.1) if the complex transfer function $H(j\omega, x)$ of the object under investigation is known:

$$u_r(t) = \text{IFT}\left[U_0(j\omega)H(j\omega, x)\right]. \quad (1)$$

Here $H(j\omega,x)$ is the complex transfer function in a pulse echo mode of the object with an arbitrary thickness x, $j = \sqrt{-1}$, $U_0(j\omega)$ is the Fourier transform of the reference signal $u_0(t)$, IFT denotes the inverse Fourier transform.

The transfer function usually is also found from pulse echo measurements using a specially prepared sample, thickness x_0 of which does not necessary is the same as the thickness of the object under investigation x:

$$H(j\omega, x_0) = \frac{U_r(j\omega, x)}{U_0(j\omega)}, \quad (2)$$

where $U_r(j\omega,x)$ is the Fourier transform of the reflected signal $u_r(t)$. Please note, that these measurements are performed in the finite frequency range $\omega_1 - \omega_2$, limited by a bandwidth of the ultrasonic transducers used in measurements.

The transfer function $H(j\omega,x)$ may be presented in terms of real $R(\omega)$ and imaginary parts $X(\omega)$:

$$H(\omega,x) = K\cdot(R(\omega)+jX(\omega)) = \\ = H_{att}(\omega,x)\cdot H_{disp}(j\omega,x), \qquad (3)$$

where $H_{att}(\omega,x) = K_0 \cdot \mathrm{e}^{-\alpha(\omega)\cdot x}$, $H_{disp}(j\omega,x) = \mathrm{e}^{-j\beta(\omega)\cdot x}$, K_0 is taking in to account signal losses and distortions caused by interfaces, $\alpha(\omega)$ is the attenuation coefficient, $\beta(\omega)$ is the propagation constant which is related to the phase velocity v_p by $\beta(\omega) = \omega / v_p(\omega)$. The transfer function of the object with an arbitrary thickness may be obtained by substituting into Eq.3 the necessary thickness x, if the frequency dependent attenuation $\alpha(\omega)$ and dispersion $\beta(\omega)$ functions are known.

Due to the limited bandwidth, a finite accuracy of manufacturing of samples and uncertainty of measurements only the attenuation coefficient $\alpha(\omega)$ may be determined with an accuracy sufficient for prediction purposes. Dispersion of ultrasonic waves even in highly attenuating materials such as plastic materials is close to uncertainty of the measurements. However, in order to predict the waveforms of ultrasonic signals, both terms of the transfer function evaluating attenuation and dispersion should be taken into account. In general, dispersion and attenuation are interrelated to each other by causal relations [5,12], therefore if there would be reliable models of attenuating media available, the dispersion could be determined from the experimentally determined attenuation function.

On the other hand, this task for many practical applications could be significantly simplified if instead of a complete attenuation function only some parameters describing the attenuation versus frequency could be exploited.

Therefore, in order to predict the waveforms of ultrasonic signals in attenuating materials with sufficient for practical applications accuracy it is necessary to solve the following tasks:

1. To estimate validity of existing models of attenuating media for materials with a high frequency dependent attenuation.
2. To develop robust measurement and signal processing procedures enabling to estimate a parameters of the frequency dependent attenuation function in plastic materials.
3. To develop calculation procedure of the frequency dependent velocity dispersion from the attenuation function estimated in the finite frequency range, using causal relations.
4. Prediction of the waveforms of ultrasonic signals in pulse echo mode taking into account attenuation and dispersion.

In order to simplify analysis the diffraction effects and the shape of boundaries of the object will not be taken into account.

First of all we shall try to find a most suitable approximation valid for description of the frequency dependent attenuation in plastic materials. The experimental technique used for estimation of the attenuation function based on pulse echo measurements is described in the next chapter.

Estimation of attenuation

The ultrasound signal losses are caused at least by two phenomena: absorption and scattering. Which one factor is dominant depends on the properties of a material and the frequency of an ultrasonic wave. Very often for analysis of ultrasonic wave propagation an attenuation function taking into account both phenomena is used. In this case it is necessary to determine the functional dependence of the attenuation versus frequency $\alpha(\omega)$. In the case of pulse echo measurements this dependence can be estimated from the ratio of the frequency spectra magnitudes of the signal transmitted through the material and the signal reflected by the front boundary of the object. The last one signal is used as the reference signal.

The attenuation and corresponding transfer function are monotonous functions of a frequency, but due to various artefacts like a limited bandwidth and influence of noise the experimentally determined functions possess an oscillating character. Also, at some frequencies spectra of the received signals possess values close to zero, therefore division of these functions may give an undefined result. So, from the practical point of view it is better to use approximation of a frequency dependent attenuation presented in an analytical form. For this purpose various approximation laws of the attenuation can be used. Usually the linear, quadratic or power law approximations are exploited [12]. For plastic materials the best results are obtained using the power law approximation [4,12,13,15]. In this case the frequency dependent attenuation can be expressed in dB by $\alpha(f) = \alpha_0\left(\frac{f}{f_0}\right)^n$, where α_0 is the attenuation coefficient in dB at the frequency f_0 [1,2,5,6,10]. The coefficients α_0 and n are defined by properties of the material. For calculation of these coefficients it was assumed that signals are affected only by attenuation and there is no dispersion. In this case the transfer function of the material becomes

$$H^{-1}{}_{att}(x,f) = \frac{U_0(f)}{U_1(f)} = K\cdot \mathrm{e}^{k\cdot x\cdot a_0\left(\frac{f}{f_0}\right)^n}, \qquad (4)$$

where $U_0(f)$ is the frequency spectra of the reference signal $U_0(f) = \left|\mathrm{FT}(u_0(t))\right|$ and $U_1(f)$ is the signal transmitted through the object $U_1(f) = \left|\mathrm{FT}(u_1(t))\right|$, k=0.115. Here K=1/K_0, K_0 is the reflectivity coefficient [2,9].

Then assuming that the measurements were performed on the sample of the known double thickness of the wall x=x_0, the coefficients α_0 and n may be found from the values of the transfer function at two different frequencies f_1, f_2. It is necessary to point out that accuracy of the approximation depends on the behaviour of the attenuation function in the vicinity of these frequencies. Therefore, the frequencies f_1, f_2 are selected according to the following criteria:

- They must be inside the bandwidth of the both ultrasonic signals;
- The attenuation function at these frequencies should possess a local minimum or at least there should be no local maximum.

The last requirement is based on an assumption that the smallest uncertainty of measurements is obtained at frequencies where the amplitude of the reflected signal is big enough.

Substitution of these frequencies into Eq.4 leads to the system of two equations [3]:

$$\begin{cases} H_1 = H^{-1}{}_{att}\Big|_{\substack{x=x_0\\ f=f_1}} = K\cdot e^{k\cdot x\cdot\alpha_0} \\ H_2 = H^{-1}{}_{att}\Big|_{\substack{x=x_0\\ f=f_2}} = K\cdot e^{k\cdot x\cdot\alpha_0\left(\frac{f_2}{f_1}\right)^n}, \end{cases} \quad (5)$$

from which the coefficients α_0 and n can be found:

$$\alpha_0(f_1) = \frac{\ln(K_0\cdot H_1)}{x},$$

$$n = \frac{\ln\ln(K_0\cdot H_2) - \ln\ln(K_0\cdot H_1)}{\ln(f_2) - \ln(f_1)}. \quad (6)$$

Such an approach was verified experimentally. Schematic presentation of the measurement equipment used for investigation of the attenuation function in a pulse echo mode is given in Fig.2.

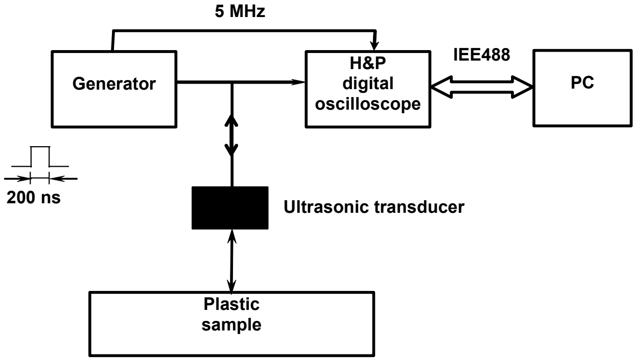

Fig.2. Experimental set-up for measurement of the attenuation coefficient using pulse echo technique

As the object for investigation was selected a single layer PVDF plastic tube with the inner diameter 111mm, the outer diameter 125mm and thickness of the wall 6.5mm. To minimize the phase distortions a focused transducer was used, which was placed at the focal distance from the front surface of the tube. For this purpose the Panametrics V309 transducer with the 5MHz central frequency, the diameter 12mm and the focal length 36.5mm was selected. The focal spot of the transmitting beam was 3mm. Data acquisition was performed with the HP54645A digital oscilloscope. The data were transferred to a personal computer via IEEE488 interface for a further analysis.

The raw signal obtained using the described experimental set-up is presented in Fig.3a. It can be seen that transmitted trough the object signal $u_1(t)$ is strongly attenuated and distorted. These distortions first of all are caused by a strong frequency dependent attenuation at higher frequencies. The modulus of the frequency spectra presented in Fig.3b shows that signal components at

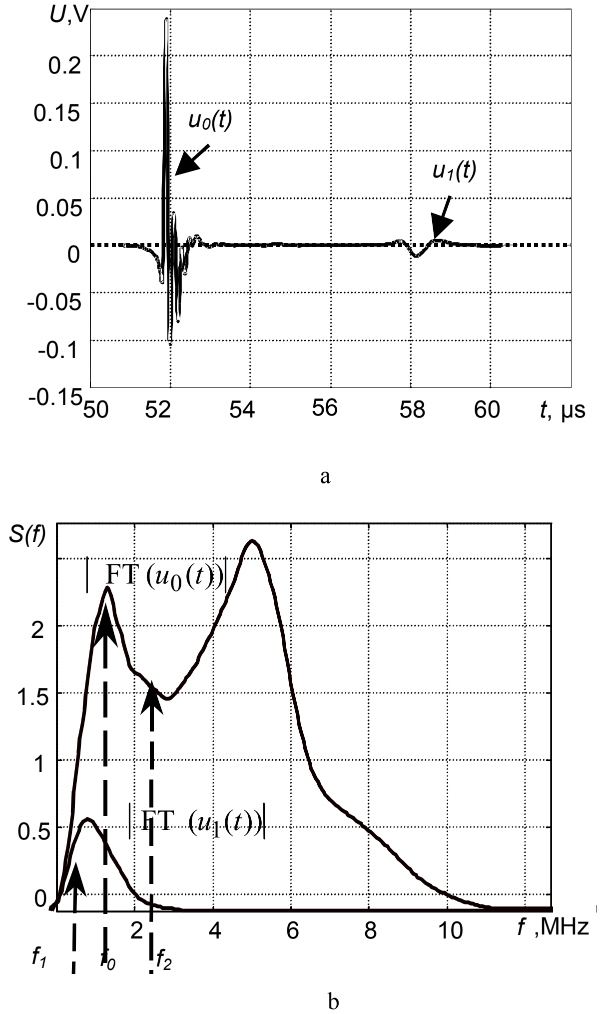

Fig.3. Calculation of the reference attenuation coefficient α_0: a-reflected signals in the time domain, b-amplitude spectra of the reflected signals

frequencies higher than 2 MHz are almost completely suppressed. Also in Fig.4 it is possible to observe the above-mentioned oscillating character of the experimentally determined transfer function. The smooth approximating function found using the proposed approach is shown by the dashed line. The frequency dependent attenuation function of ultrasonic waves in PVDF plastic material is presented in Fig.5. The coefficients α_0 and n determined at the frequency f=1.45MHz are α_0=17.66dB/cm and n=1.17.

Estimation of dispersion

The technique used to estimate ultrasound velocity dispersion from the measured attenuation in a particular material is based on causal relations [5,12]. The first steps attempts were made in the field of electromagnetic waves and were exploiting causality of the Kramers-Kronig relations for phase velocity and amplitude attenuation. It was proposed that for a unbounded attenuated medium are valid the following conclusions [5]:

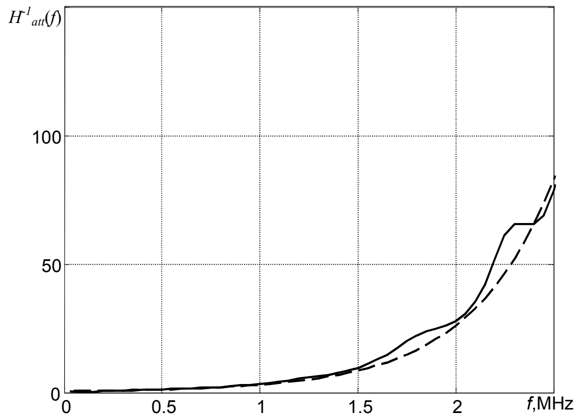

Fig.4. Transfer functions of PVDF plastic layer in a pulse echo mode: solid line-experimental results, dashed line-approximation $K \cdot e^{k \cdot x \cdot \alpha_0 \left(\frac{f}{f_1}\right)^n}$

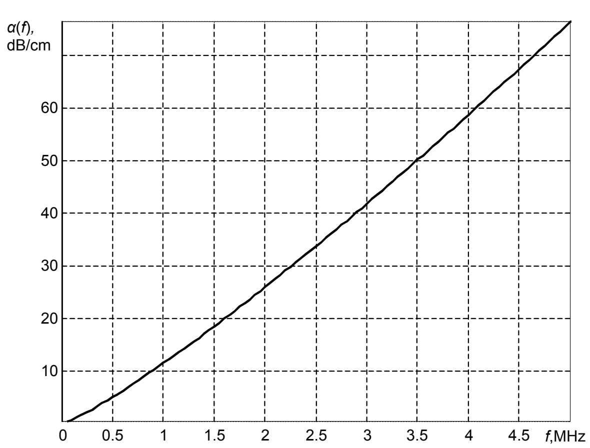

Fig.5. Experimentally determined frequency dependent attenuation coefficient of ultrasonic waves in PVDF type plastic material

1. The phase velocity $v_p(\omega)$ of the wave is frequency dependent (dispersion).
2. The attenuation coefficient $\alpha(\omega)$ is frequency dependent and increasing with frequency.
3. The phase velocity and attenuation coefficients are not independent, but are interrelated and can be determined each from other.

If there is no sharp resonance over the frequency range of investigation, the relation between ultrasound velocity and attenuation are given by [5,7]:

$$\alpha(\omega) = \frac{\pi \cdot \omega^2}{2 \cdot v_p^2(\omega)} \cdot \frac{dv_p(\omega)}{d\omega}, \tag{7}$$

$$\frac{1}{v_p(\omega_R)} - \frac{1}{v_p(\omega)} = \frac{2}{\pi} \int_{\omega_0}^{\omega} \frac{\alpha(\omega')}{\omega'^2} d\omega', \tag{8}$$

where ω_R is the reference frequency, $\omega_0 = 2\pi f_0$. These simplified expressions are obtained from linearity and causality requirements and are generally known as Kramers-Kronig relationship. They are important, because enables to determine the dispersion from the attenuation versus frequency and vice versa. These relationships were validated for acoustic waves in various materials experimentally [5,6,7,13,14].

The first implementation of these relations was based on the Hilbert transform, but it has a serious shortcoming, because it is necessary to know attenuation coefficient values in all range of the frequencies [11]. Therefore, for our analysis we have selected nearly local (n<3) and time causal (0<n<1, 1<n<2) forms of the Kramers-Kronig relationship. The advantage is that we can calculate frequency dependent ultrasound velocity dispersion from the estimated attenuation coefficient measured at the one reference frequency value f_R [6,8,14].

In the time causal form approach, when $n \neq 1$, the difference of the phase velocities can be calculated as [6,8,12,13]

$$\frac{1}{v_p(f_R)} - \frac{1}{v_p(f)} = \frac{1}{\Delta v_{TC}(f)}, \tag{9}$$

where $$\frac{1}{\Delta v_{TC}(f)} = -(2 \cdot \pi)^n \cdot \frac{\alpha_R}{2 \cdot \pi \cdot f_R^n} \times \tan\left(\frac{n \cdot \pi}{2}\right) \cdot \left(f^{n-1} - f_R^{n-1}\right).$$

In the nearly local form approach, the difference of the phase velocities is given by [5,6,8]:

$$\frac{1}{v_p(f_R)} - \frac{1}{v_p(f)} = \frac{1}{\Delta v_{NL}(f)}, \tag{10}$$

where $$\frac{1}{\Delta v_{NL}(f)} = (2 \cdot \pi)^n \times \frac{\alpha_R}{f_R^n \cdot \pi^2 \cdot (n-1)} \cdot \left(f^{n-1} - f_R^{n-1}\right).$$

The total phase velocity dispersion can be determined from Eq.11 [14]:

$$v_p(f) = \frac{v_p(f_R)}{1 - \frac{v_p(f_R)}{\Delta v(f)}}. \tag{11}$$

where α_R is the reference attenuation coefficient (dB/cm), $v_p(f_R)$ is the reference ultrasound velocity value (m/s) in the sample at the single frequency f_R (Hz), $\Delta v(f) = \Delta v_{TC}(f)$ for the time causal method and $\Delta v(f) = \Delta v_{NL}(f)$ for the nearly local method approach.

A very essential question is how the reference frequency f_R must be selected. There are two criteria for that. The first one is that the reference frequency must be inside the frequency bandwidth of the signal transmitted through the object $u_r(t)$. The second is that due to the accuracy requirements it must correspond to the frequency at which the transmitted signal possesses maximum spectrum amplitude. As it was mentioned above the magnitude of the signal frequency spectrum of the signal $u_r(t)$ usually has a complicated shape, therefore the best approach for the determination of f_R is to apply the second order autoregressive models in the frequency range [f_1,f_2] [16].

For the described above sample of a plastic tube with a wall thickness x_w=6.5mm and the measured ultrasound velocity v_p=2135m/s the reference frequency was selected f_R=1.45MHz. The attenuation coefficient at that frequency calculated according to the chosen approximation was α_R=17.66dB/cm. The calculated ultrasound velocity dispersion curves for the nearly local and the time causal models are presented in Fig.6. It can be seen that both models give very similar results. It can be explained by the fact that for the investigated sample the power of frequency dependent attenuation approximation n is close to 1, that is n=1.17. It is known that in such cases time causal and nearly local models are close to each other [5].

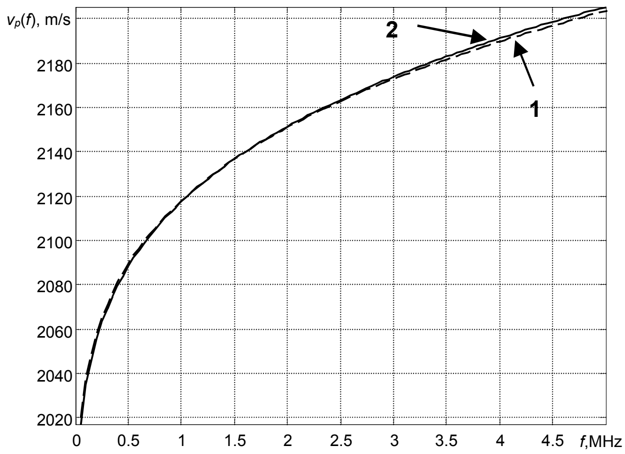

Fig.6. Ultrasound velocity versus frequency in PVDF plastic material: 1- time causal model, 2- nearly local model

Prediction of the ultrasonic waveforms

The prediction task may be split into two separate subtasks: prediction of ultrasonic waveforms taking into account only a frequency dependent attenuation of ultrasonic waves and prediction taking into account both the attenuation and dispersion phenomena. The first approach is attractive from the point of a view of practical applications, because the prediction requires less *a priori* information about acoustic properties of the material. We shall investigate both approaches in order to find out what discrepancies are obtained in each case.

In both cases prediction is performed by filtering of the reference signal in the frequency domain by the filter with the medium transfer function $H(j\omega,x)$. For the first simplified approach the transfer function is given by $H(j\omega,x)=H_{att}(\omega,x)$ and for the second approach $H(j\omega,x)=H_{att}(\omega,x)H_{disp}(j\omega,x)$. Strictly speaking, in the case of the simplified approach the causality condition is not fulfilled, but if we are not interested in the absolute delay of the signals, then it may be applied. The signal reflected by the back wall of the object with the thickness x_w can be calculated using the following expression:

$$u_{1c}(t) = -\mathrm{IFT}\left(K_0 \cdot \mathrm{FT}(u_R(t)) \cdot \mathrm{e}^{-2\cdot k\cdot x_w\cdot \alpha(f)}\right) = -\mathrm{IFT}\left(K_0 \cdot \mathrm{FT}(u_R(t)) \cdot e^{-2\cdot k\cdot x_w\cdot \alpha_R \cdot \left(\frac{f}{f_R}\right)^n}\right). \quad (12)$$

where FT and IFT denote the direct and inverse Fourier transform respectively.

When the velocity dispersion is taken into account the predicted signal is given by [17]:

$$u_{2c}(t) = -\mathrm{IFT}\left(K_0 \cdot \mathrm{FT}(u_R(t)) \times \mathrm{e}^{-2\cdot k\cdot x_w\cdot \alpha_R \cdot \left(\frac{f}{f_R}\right)^n - j\cdot 2\cdot \pi\cdot x_w \frac{f}{v_p(f)}}\right), \quad (13)$$

where $v_p(f)$ is the frequency dependent phase velocity.

The predicted signals reflected by the back wall of sample of the plastic pipe are presented in Fig.7. The experimentally measured signal is presented in Fig.7 by the solid line. It can be seen that similarity between the measured signal and the signal predicted using the complete model is better then in the case when only frequency dependent attenuation was taken into account. The calculated correlation coefficients between the measured and predicted waveforms for both analysed cases are correspondingly 0.993 and 0.926.

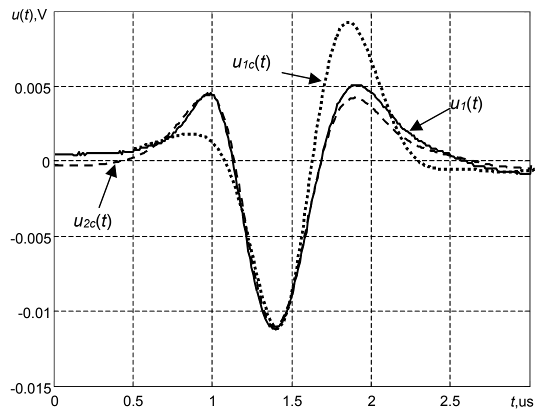

Fig.7. Waveforms of the attenuated signals: $u_1(t)$- measured, $u_{1c}(t)$- predicted taking into account only attenuation, $u_{2c}(t)$- predicted taking into account attenuation and velocity dispersion

Conclusions

The results presented illustrate that the approach proposed enables to predict waveforms of ultrasonic signals in highly attenuating materials like plastics with a reasonable for practical applications accuracy. The prediction accuracy is obtained better when the ultrasound velocity dispersion is taken into account.

Robustness of the developed procedure in the future will be checked comparing the predicted signals and results of experiments carried out at different points of the sample, using for prediction the attenuation and velocity dispersion functions measured only at one specified point. That would enable in the future for prediction to exploit a database, containing attenuation functions, created for a particular class of materials.

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Ultragarsinių signalų, sklindančių smarkiai slopinančiais plastikais, laikinės formos apskaičiavimas

Reziumė

Straipsnyje pateiktas slopinimo ir dispersijos dažninių priklausomybių nustatymo smarkiai slopinančioje medžiagoje metodas. Slopinimo dėsnis nustatomas atliekant perdavimo funkcijos aproksimaciją dažnio srityje. Ultragarso greičio dispersija skaičiuojama iteraciniu priartėjimo būdu panaudojant Kramerio ir Kronigo priežastingumo principą ir žinomas ultragarso greičio ir medžiagos storio reikšmes.

Pateiktas metodas įgalina atkurti laikinę atspindėto signalo formą, įvertinant daugkartinius atspindžius. Pateikti eksperimentiniai pasiūlyto metodo taikymo plastikinių vamzdžių parametrams matuoti, rezultatai.

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