Determination of piezoceramics parameters by the use of mode interaction and fitting of impedance characteristics

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Introduction

Parameters and characteristics of piezoceramic based ultrasonic transducer are decisive for performance of ultrasonic scanners and other ultrasonic devices. Modern design technologies of transducers require knowledge of practically all parameters of piezomaterial: compliance or stiffness, piezoelectrical coupling and permittivity. A specification given by producers often doesn't include all necessary parameters and/or accuracy of parameters is not sufficient. One reason of insufficient accuracy is inevitable dispersion of mechanical properties, e.g. density. It could occur not only in different piezosamples but also in the same piece of piezoceramic (so called "cloudiness" of properties). The other reason is different conditions of polarization technology, e.g. different local temperature. When the small pieces of piezoceramic are shaped from the bigger one, the dispersion of parameters due to inhomogenity becomes evident. A commercial producer usually doesn't perform measurements for each sample but presents highly averaged parameters. Therefore it is a need to test each piezoelement before producing a transducer with precisely predefined characteristics.

Piezoelements of modern transducers are prepared using novel technologies of brushing, coating and polarization. Those procedures can change parameters of piezomaterial significantly in volume and under electrodes and also can influence overall characteristics of piezotransducer. Transducers, especially multielement arrays of ultrasonic scanners are composed from many piezoelements (up to several hundred), which should have as identical parameters as possible. Proper design of transducer requires accurate knowledge of all main parameters of piezomaterial and is in fact individual for each transducer, when it comes to the transducer of highest quality.

One of the widely-used and most advanced tools for transducer design and simulation is a finite element method (FEM) [1], [2]. The FEM packages like ABAQUS, PZFlex or ANSYS [3] are suitable for analysis of piezoelements.

Reliability of results of FEM simulations depends not only on adequacy of model itself but also on adequacy of material parameters. The main source of disagreement between results of simulation and measurements is insufficient adequacy of experimentally defined parameters of piezoceramic and those used for FEM simulations [4].

There are several methods to overcome existing problem. The first suggestion is to measure all necessary parameters manually. This approach requires at least 5 different shapes of piezoelement in order to ensure pure vibration modes [5], [6]. It is substantially complicated and time consuming. Usually a piezoelement could not be used for transducer production after measurements because shape of initial piezosample is corrupted due the shaping of 5 different piezoelements. Therefore the problem exists in evaluation of parameters of piezosample which has quite arbitrary shape (e.g. most common disc shape samples).

To overcome limitations of classical methods a set of parameters should be determined from one piezoelement and possibly at one frequency range. This goal can be achieved when the maximal quantity of modes are piezoactive. Such situation occurs when the modes are interacting with each other. This phenomenon is called mode interaction or mode coupling.

The frequencies of pure modes could be calculated according well-known equations [5], [6], [7]. Mode interaction exists in vibrating piezoelement having such geometrical shape when the main frequencies or harmonics of different modes are in the same frequency range. In the case of mode interaction the frequencies of resonance differs from pure mode frequencies.

There are a limited number of analytical methods to calculate mode interaction. In case of radial and thickness mode coupling this can be done according [8]. But this approach isn't suitable for practical reasons because it requires a set of parameters which differs from IEEE recommendations [6]. Therefore special methods for measurements are required [9].

For evaluation of mode interaction more flexible approach is computational. The main advantage of computational methods, for example FEM, is versatility.

Mode interaction analysis by FEM

Mesh grid size and frequency errors

When the interaction exists between thickness ant transverse (radial) vibrations, the 2D model can be used for FEM analysis. It saves time and resources of computation. Another time-consuming point is FEM mesh grid size. Model complexity and computation time depends on element size h in FEM model. On the other hand, reliability of calculations depends on elements per

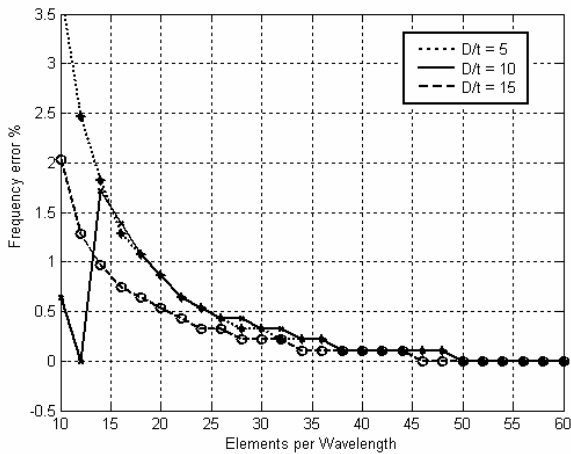wavelength λ/h. This rate usually is 10 to 20 and accuracy of calculations is from 3% to 1% [10].

Fig. 1. **Dependence of frequency error on number of elements per wavelenght (λ/h)**

For higher precision an additional analysis of grid size was performed. It is important for piezoelement with mode interaction because the stronger the coupled mode, the higher λ/h is required for adequate simulations.

A disk shape piezoelement was selected for analysis because in this type of piezovibrator two main modes – radial and thickness – are piezoactive. Another advantage of this shape is possibility to control mode interaction by changing dimensions: diameter D ant thickness t. Strength of interactions depends on ratio D/t: the less this ratio, the stronger is mode interaction.

FEM model was built using 2D axisymmetric approach. The influence of FEM grid size for frequency errors was analyzed. This influence is different for radial and thickness modes [11]. It was observed also, that a higher λ/h is required for higher level of mode interaction. When the modes interaction phenomenon is under investigation, $\lambda/h>25$ is recommended (Fig.1). Therefore grid size was selected equal to 25 finite elements per wavelength for upper frequency of analysis (1.4 MHz). It is obvious that for lower frequencies condition $\lambda/h>25$ will be valid always.

Mode interaction and frequencies of radial and thickness resonance

Each piezoceramic has own set of parameters: stiffness (c^E_{11}, c^E_{12}, c^E_{13}, c^E_{33}, c^E_{44}), piezoelectrical coupling (e_{31}, e_{33}, e_{15}) and permittivity (ε^S_{11}, ε^S_{33}). The mode interaction depends on almost all of them and it depends on proportion between them also. Therefore shape of impedance characteristic highly depends on ceramic type.

A PZT based piezoceramics (CTS-19, which is similar to PZT-4) was selected for investigation of radial and thickness mode interaction.

The radial and thickness modes could be analyzed in two aspects – electrical and mechanical. Electrical aspect means impedance or admittance characteristics of piezoelectrical plate; mechanical means movement of surface of piezoelement. In our case the impedance modulus frequency dependence $Z(f)$ of piezoelement without acoustical load was calculated.

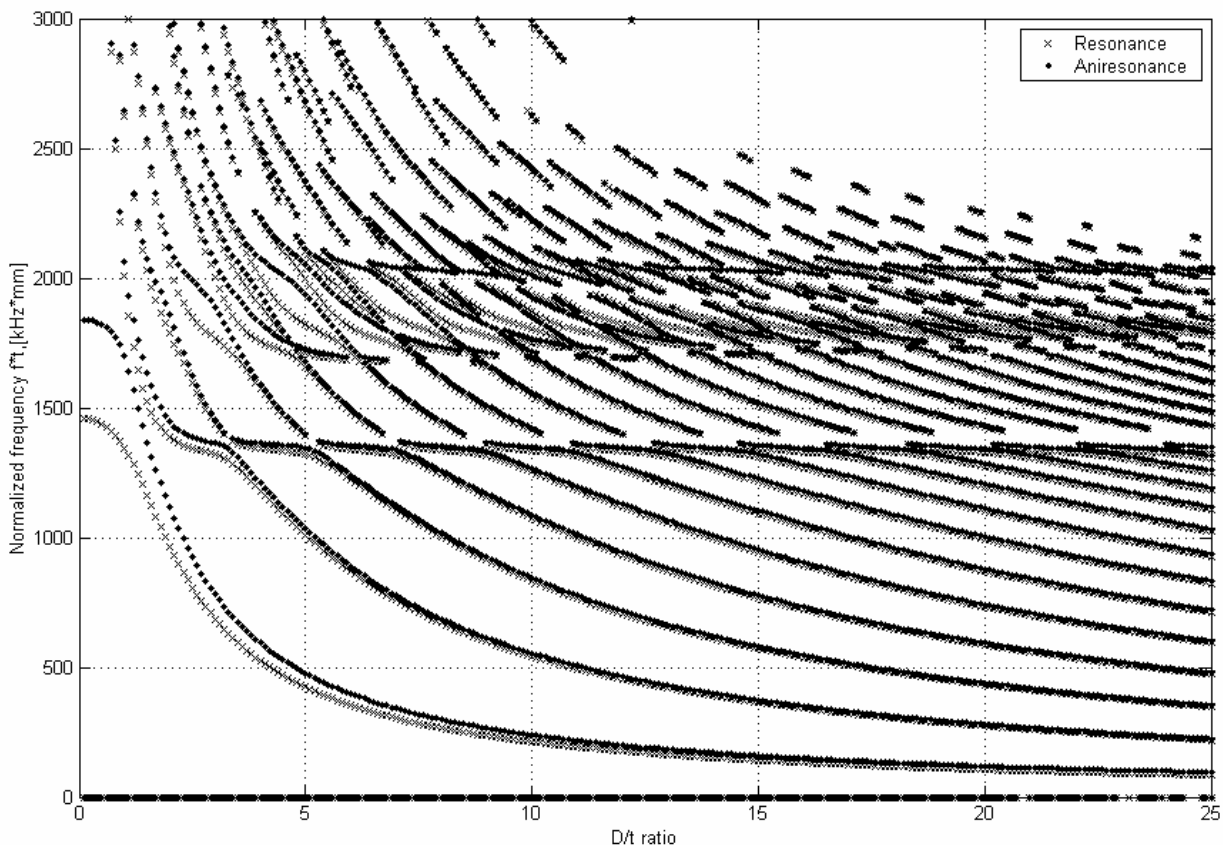It is known that mode interaction changes the

Fig. 2. **Frequencies of modes in disk shape piezoelement**

resonance and antiresonance frequencies of radial mode [12]. For thickness mode it changes not only the frequencies but also makes the shape of Z(f) quite complicated. Therefore an influence of radial mode to the frequency of thickness resonance was analyzed. Thickness t was left constant ($t=1.6$ mm) and diameter D was varied owre wide range.

On the other hand, Z(f) doesn't represent ceramic, it represents characteristic of piezoelement of given thickness. A normalized frequency (f·t) was used for

Fig. 3. **Impedance characteristics for various D/t ratios**

characterization of ceramic impedance. The full

representation of Z(f·t) for wide range of D/t should be visualized as a three-dimensional surface Z(f·t, D/t). Though the difference in amplitude of Z is very high, especially for higher ant lower values of D/t, therefore 3D representation isn't picturesque. Two dimensional representation of frequencies of resonance and antiresonance is more suitable for analysis of mode interaction (Fig.2).

The additional information, which isn't represented in Fig.2, is magnitude of Z. A shape of Z(f·t) can be used for rough visual evaluation of mode interaction. The shape of this characteristic near thickness mode is sensitive for mode interaction (Fig.3). A pure mode (D/t=25) has one-peak characteristic, while a mode influenced by radial modes (D/t=10) usually has few peaks in Z(f·t) and is complicated in shape. In case of high level of interaction between two modes (D/t=2.5), the antiresonance peak of thickness mode has quite significant frequency shift. High sensitivity of the impedance shape to the mode interaction as well as complicated multipeak curve opens possibility for precise fitting of piezomaterial.

Impedance fitting method

Optimization task

All of parameters of piezoceramic have an influence to the frequencies of impedance of different modes. This influence changes not only frequencies but also whole shape of impedance curve as it was noted above. When the radial and thickness mode interaction occurs, it's quite difficult to determin influence of each parameter, especially in lossy ceramic. It can be estimated by influence to overall shape of impedance curve.

The main task is more accurate evaluation of each piezoparameter by minimization of differences between two impedance characteristics: curve under investigation and basis-curve. Curve under investigation is FEM calculations result. Basis-curve may represent experimental curve. When these characteristics are made close (by fitting of piezomaterial parameters), then differences of both sets of parameters are minimal. Difference between two characteristics can be minimized using optimization algorithm. One of goals of present study was a development of method for parameter fitting by the use of optimization. Therefore reference basis-curve was calculated by the use of FEM keeping recommendations for grid size. Such curve adequately simulates experimental results.

According to optimization task, an objective function (goal function) and optimization variables should be defined. The optimization variables are all parameters of piezoceramic.

Objective function

Comparison of two impedance characteristics can be done in different ways, based on least squares fitting methods: least mean square, nonlinear inversion [13], etc. In present work it was developed a method which is based on correlation between two characteristics.
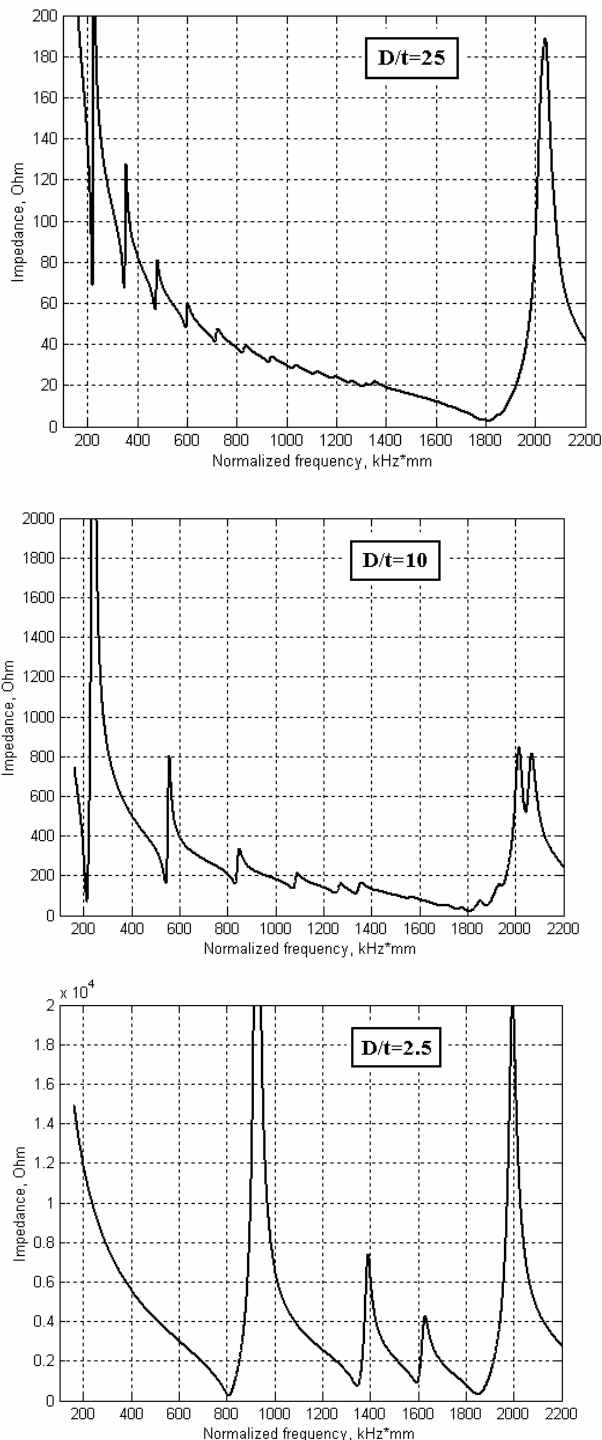
The core of this method is a coefficient of correlation between curve under investigation (ci) and basis-curve (cb):

$$CC = \frac{n \cdot \sum_n ci \cdot cb - \sum_n ci \cdot \sum_n cb}{\sqrt{\left[n \cdot \sum_n ci^2 - \left(\sum_n ci \right)^2 \right] \cdot \left[n \cdot \sum_n cb^2 - \left(\sum_n cb \right)^2 \right]}}, \quad (1)$$

where n is number of data points in characteristics. Coefficient of correlation can indicate a similarity between two characteristics. It was found [11], that shapes of two characteristic are approximately similar, when $CC > 0.9$. Then a difference of frequency for each peak in $Z(f)$ is less than 0.5%.

In our case objective function can be defined as

$$OBJ = 1 - CC. \quad (2)$$

In order to fit two impedance curves the objective function should be minimized by optimization process.

Influences of separate parameters

The set of parameters of piezoceramic for optimization can be properly chosen when the influence of each parameter on objective function (OBJ) is known. Also it is necessary to determine frequency range in which this optimization should be performed for maximal sensitivity of maximal quantity of parameters.

The sensitivity of OBJ to the each parameter of piezoceramic was analyzed for three piezoelectrical disks with different aspect ratios: D/t = 2.5; 10; 25. In other words these calculations were performed for different levels of mode interaction – strong, medium and low.

FEM analysis was split into two frequency ranges. The sensitivity analysis was performed in frequency zones of first radial mode and thickness mode respectively. This approach allowed estimation of influence of different parameters to different modes and estimate different level of interaction.

Influence of separate parameter was evaluated by sweeping this parameter in the range +/- 10% over initial value. All other parameters were set at constant initial values. Following this approach, 10 independent sweep calculations for each piezoelement and each frequency range is required. Alternative method is calculation of derivatives at initial values of parameters but it could be useful in narrow range of parameter variation.

The results of sensitivity analysis are presented in Fig.4a - Fig.4c. All stiffness parameters are separated from piezoelectric and dielectric parameters seeking for better visualization of results. The attention should be paid to different scale of axis.

A detailed analysis of sensitivity graphs presented indicate a cases where maximal quantity of parameters has significant influence to shape of $Z(f \cdot t)$.

Analysis of sensitivity

It can be noticed that dielectric permittivity ε^S_{11} has no influence to the $Z(f \cdot t)$ in all cases. Therefore it isn't usable for fitting algorithm.

For the three aspect ratios of piezoelectrical disk the lowest quantity of parameters can be determinated from single mode vibrations (Fig.4a). When D/t is high enough (D/t = 25), stiffness c^E_{44} and piezoelectric coupling e_{15}
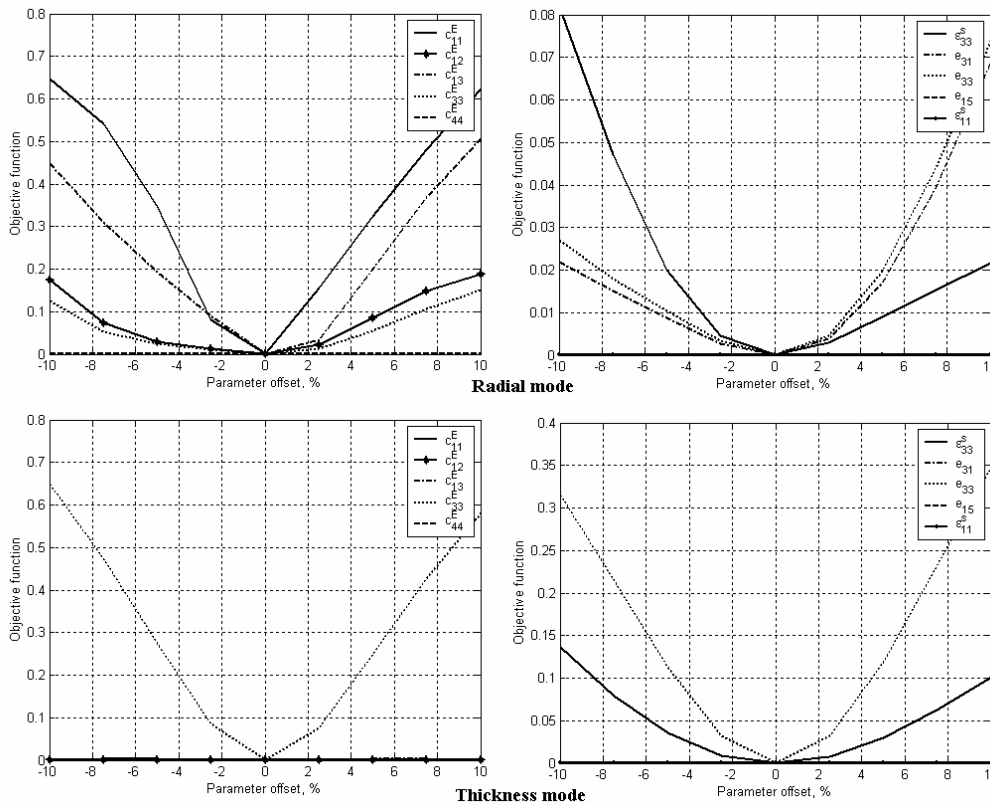

Fig. 4a. **Sensitivity of objective function to parameters of piezoceramic (D/t = 25)**
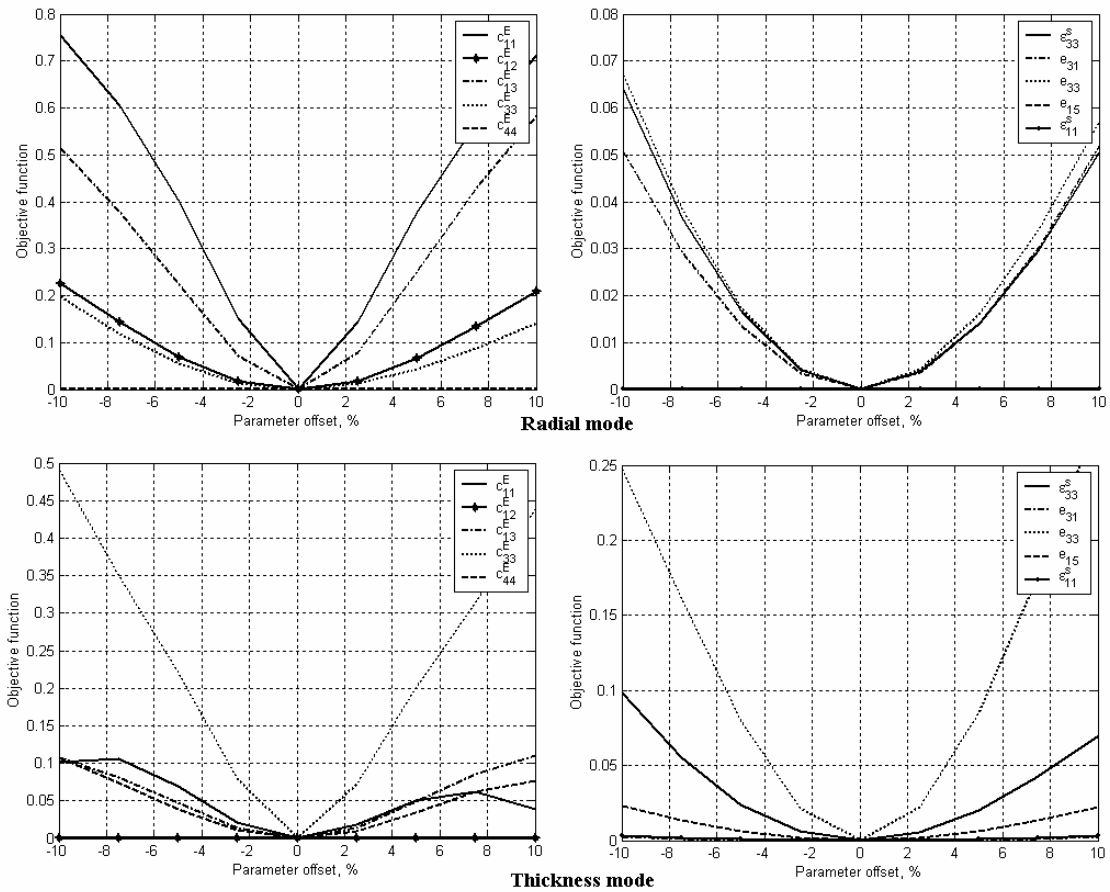
Radial mode

Thickness mode

Fig. 4b. **Sensitivity of objective function to parameters of piezocera4mic (D/t = 10)**

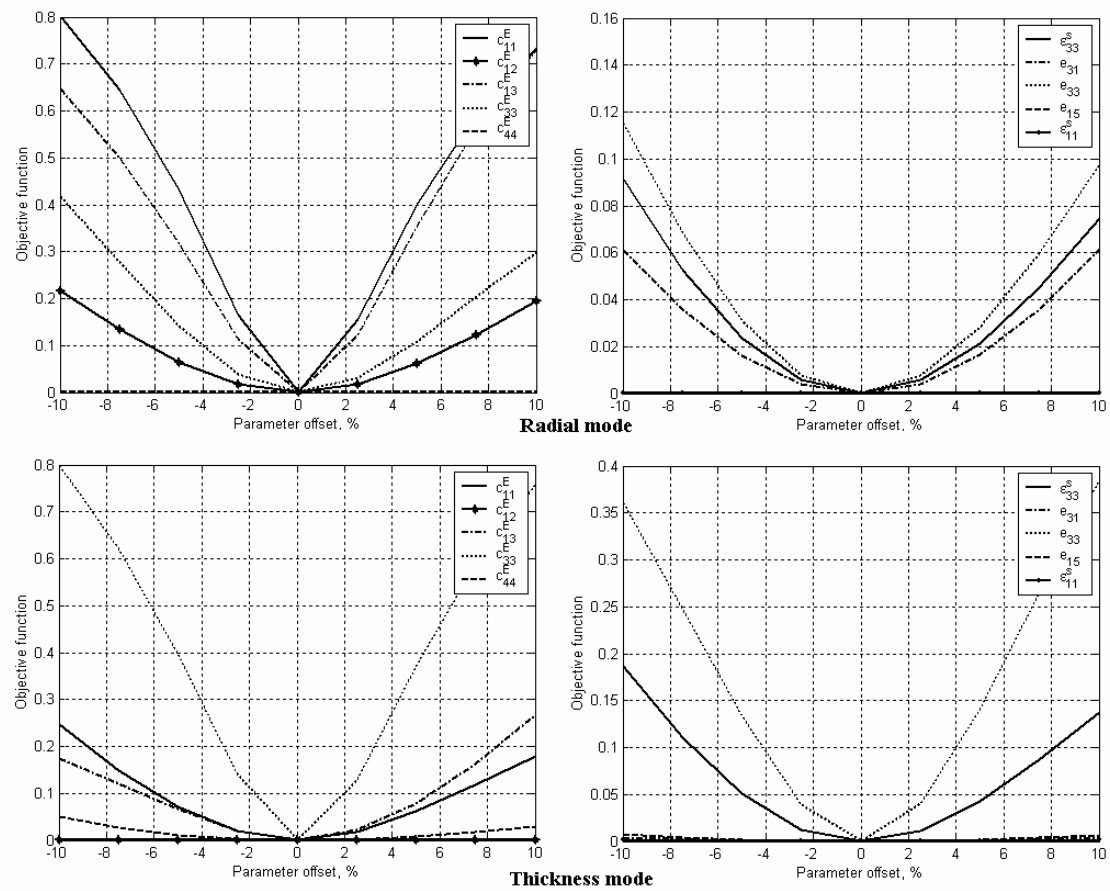

Radial mode

Thickness mode

Fig. 4c. **Sensitivity of objective function to parameters of piezoceramic (D/t = 2.5)**

couldn't be used as optimization variables. Determination of all other parameters requires calculation of $Z(f \cdot t)$ over a wide range of frequency.

The impedance characteristics of piezoelements with mode interaction are more sensitive to the all set of parameters. The highest sensitivity is achieved in piezoelement with aspect ratio $D/t = 2.5$. On the other hand, the medium level mode interaction ($D/t = 10$) allows determination of piezoelectrical coupling parameter e_{15}. Therefore this aspect ratio was selected as main for $Z(f \cdot t)$ fitting. In order to prove this decision, $D/t = 2.5$ disk was used as auxiliary with the same conditions of optimization.

Frequency range of calculations is also important.. In order to use some advantages of radial mode sensivity to stiffnes parameters, lower analysis frequency of the range was expanded. Thus a some harmonics of radial mode were included into analysis. This increases the fitting accuracy of parameters, responsible for radial vibations of piezoelement

Fitting of piezoceramic parameters

Two-level optimization algorithm was used for the fitting of impedance characteristics. The first level of optimization is a random design set generation. Parameters were randomized over specified range and the values of objective function were calculated.

Second level is based on ANSYS built-in subproblem approximation method which is a zero-order method of optimization [3]. This method requires only values of optimization variables and objective function but not their derivatives. Values of objective function for different sets of parameters were approximated by curve. The approximation method was set as quadratic and cross-term. This approximation was updated after every iteration of optimization.

Values of objective function and optimization variables from first level of optimization are data point for approximation in second level. Then further optimization is performed according subproblem approximation algorithm.

The initial parameters for optimization couldn't obtain whatever value because the stiffness matrix should be positively defined for proper work of FEM equation solver. The values of piezoparameters in optimization process were constrained in the range +/- 10% over original values. In this range stifness matrix for CTS-19 ceramic remains positively defined. (In the case of other piezomaterial or other parameter variation range constraint of positively defined sifness matrix should be checked additionaly). The initial values before optimization were "detuned" by -10%.

The same initial conditions of optimization was used for both piezoelelements ($D/t = 10$ and $D/t = 2.5$).

Results of parameters fitting are different for either piezoelement (Table 1). Better results was in case of medium level of mode interaction ($D/t = 10$) as it was predicted in sensitivity analysis.

The impedance characteristic which corresponds to initial set of optimization parameters is different in frequencies of thickness and radial modes (Fig.5-a). When the optimization is performed (Fig.5-b) all the frequencies are fitted. The main residual error is in magnitude of impedance characteristic.

Table 1**. Parameters of piezoceramic in process of optimization**

Parameter	Original value	D/t = 10		D/t=2.5	
		Value after optimization	Error, %	Value after optimization	Error, %
c^E_{11}, Pa	$10.9 \cdot 10^{10}$	$10.977 \cdot 10^{10}$	0.706	$10.557 \cdot 10^{10}$	3.15
c^E_{12}, Pa	$6.1 \cdot 10^{10}$	$6.103 \cdot 10^{10}$	0.049	$6.195 \cdot 10^{10}$	1.55
c^E_{13}, Pa	$5.4 \cdot 10^{10}$	$5.547 \cdot 10^{10}$	2.722	$5.272 \cdot 10^{10}$	2.36
c^E_{33}, Pa	$9.3 \cdot 10^{10}$	$9.432 \cdot 10^{10}$	1.419	$9.241 \cdot 10^{10}$	0.63
c^E_{44}, Pa	$2.4 \cdot 10^{10}$	$2.422 \cdot 10^{10}$	0.916	$2.489 \cdot 10^{10}$	3.72
e_{15}, C/m^2	10.6	10.674	0.698	10.479	1.14
e_{31}, C/m^2	-4.9	-4.947	0.959	-5.096	4
e_{33}, C/m^2	14.9	15.226	2.456	15.777	5.88
$\varepsilon^S_{11}/\varepsilon_0$	826	NA	NA	NA	NA
$\varepsilon^S_{33}/\varepsilon_0$	840	923	9.88	922.8	9.85
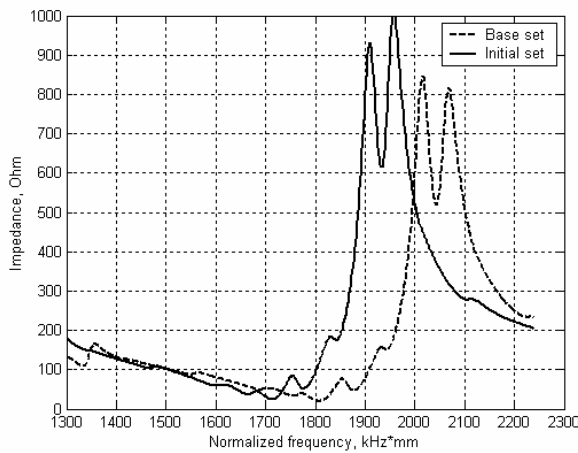
a)
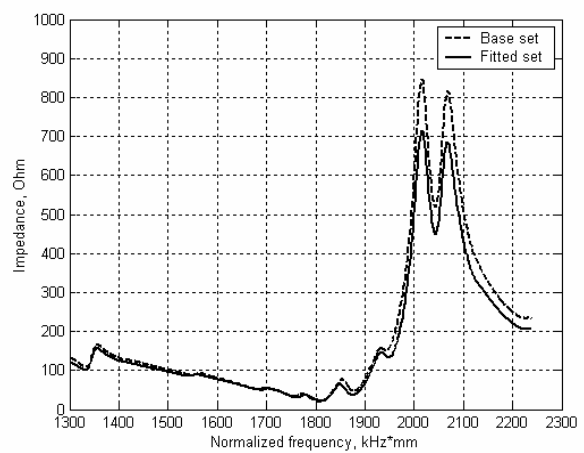
b)

Fig. 5. **Impedance characteristic before (a) and after (b) optimization (D/t = 10)**

Discussions

The presented method is based on impedance characteristics fitting by change a set of parameters of piezoceramic. This method can return initially detuned values of piezoparameters to their initial values with tolerance 1-3%. Fitting method can be useful when standard methods are applied for measurement of parameters of ceramics in piezoelement with medium mode interaction (when samples don't ensure truly pure-mode vibrations). The errors in determination of parameters then could be decreased by impedance fitting procedure presented.

The only parameter which wasn't fitted is ε^S_{33}. This is because influence of permittivity to the magnitude of impedance is more significant than influence to the frequencies [5]. An assumption that permittivity can be fitted in pure thickness mode requires further investigation.. On the other hand, permittivity can be measured directly and there is no need for optimization of ε^S_{33} in wide range.

Some residual differencies in magnitudes of original and fitted characteristics are caused by insensitivity of correlation coefficient to magnitudes of two data sets. Therefore an additional term sensitive to magnitude, could be added to the objective function.

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Pjezokeramikos parametrų nustatymas panaudojant modų sąveiką ir impedanso charakteristikų sutapdinimą

Reziumė

Matuojant pjezokeramikos parametrus (pjezopastoviąsias) įprastais metodais, modų sąveika yra nepageidaujamas reiškinys. Priklausomai nuo stiprumo ji keičia būdinguosius dažnius ir mažina parametrų nustatymo tikslumą. Straipsnyje pateikiamoje metodikoje modų sąveika naudojama pjezokeramikos parametrams patikslinti. Ši metodika remiasi baigtinių elementų metodu bei daugiaparametriu optimizavimu pagal pjezoelemento impedanso charakteristikų sutapimą. Tikslo funkcija pasirinkta atraminės ir apskaičiuotos impedanso kreivių tarpusavio koreliacija. Siekiant padidinti impedanso kreivės formos jautrį kiekvienai pjezopastoviajai, panaudota modų sąveika. Pasirinkti pjezoelemeto matmenys ir tiriamųjų dažnių ribos, kur impedanso kreivės formai įtakos turi tiek storio, tiek skersinius virpesius lemiančios pjezopastoviosios. Taikant šią metodiką, pjezokeramikos parametrų nustatymo neapibrėžtį priklausomai nuo pjezopastoviosios, galima sumažinti iki 1-3%.

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