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Investigation of the oscillatory dynamics of the plate with the control layer of material

M. Ragulskis, V. Kravčenkienė

Kaunas University of Technology, Department of Mathematical Sciences

Introduction

The emerging concept of active constrained layer control of vibrating structures involved the application of a viscoelastic layer constrained by a piezoelectric constrained layer. Attempts have been made to optimize the performance of the constrained layer treatments by selecting the optimal thickness, length, elasticity of the viscoelastic core [1, 2, 3].

This paper deals with the control of dynamics of an elastic structural plate by application of a layer of material the parameters of which can be varied by selecting the thickness of the layer.

Another problem occurring in such a type of applications is the appropriate interpretation of the results of simulation. Comparisons between the vibrations of the core and the controlling surface are very important from the point of design of controlling elements, though conventional visualization techniques can produce rather poor results in that sense. Therefore there exist a definite need for developing more appropriate visualization techniques capable of detection small variations from a predefined regime of dynamic motion.

The plate as a structural element by taking into account the layer of material on its surface is analyzed. This serves as a model of the transporting element with the interacting transported material. The simplified model of the transported material consists of the distributed springs with masses performing vertical motion and without mutual interaction. This is a modification of the Winkler type elastic layer known in applied elasticity [5]. The finite element with four degrees of freedom per node (the deflection and the two rotations of the plate and the deflection of the layer) is developed for the analysis of the described systems. The eigenmodes are calculated and each of them provides the two surfaces: the first one describing bending of the core plate, another – the motion of the control layer.

Model of the plate with the layer of material

The model of the analysed system is presented in Fig.1. It can be noted that the analysed system is a continuous system in the sense of its core and control elements. Though the principal scheme in Fig. 1 involves lumped parameters for the Winkler layer and the external mass layer, it is understood that both layers are continuous. The discrete schematic visualisation can be explained by the fact, that the external mass layer is described as having no inter-element relations. Otherwise it could be interpreted as a layer of normal deformable body, but in that case it would be not applicable for investigation of the effects of dynamic conveyance. On the other hand, the Winkler stiffness layer must perform the function of an active vibration controller. That requires the possibility of dynamic stiffness variation. Conventional FEM modelling would require re-calculation of global system matrixes in every time step.

Development of a numerical model of the described system required the derivation of new element which is a modification of the general plate element presented in [4].

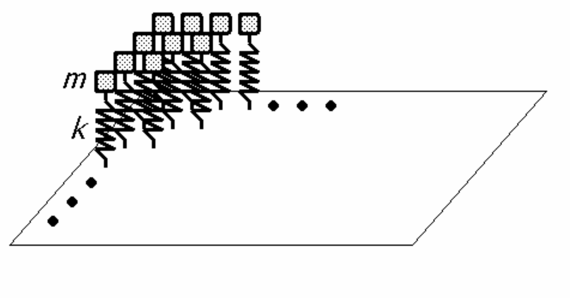

Fig. 1. **The model of the plate with the layer of material**

The nodal variables are the deflection of the plate w, the rotation of the plate about the x axis Θ_x, the rotation of the plate about the y axis Θ_y, the deflection of the layer w^* (it is assumed that $v = -z\Theta_x$, $u = z\Theta_y$).

The potential energy of the layer of the distributed springs is:

$$\begin{aligned}\Pi &= \frac{1}{2}\iint k\left(w-w^*\right)^2 dxdy = \\ &= \frac{1}{2}\iint k\left(\left[\overline{N}\right]\{\delta\}-\left[\overline{\overline{N}}\right]\{\delta\}\right)^2 dxdy = \\ &= \frac{1}{2}\{\delta\}^T\iint\left(\left[\overline{N}\right]-\left[\overline{\overline{N}}\right]\right)^T k\left(\left[\overline{N}\right]-\left[\overline{\overline{N}}\right]\right)dxdy\{\delta\} = \\ &= \frac{1}{2}\{\delta\}^T\iint\left(\begin{matrix}\left[\overline{N}\right]^T k\left[\overline{N}\right]+\left[\overline{\overline{N}}\right]^T k\left[\overline{\overline{N}}\right]- \\ -\left[\overline{N}\right]^T k\left[\overline{\overline{N}}\right]-\left[\overline{\overline{N}}\right]^T k\left[\overline{N}\right]\end{matrix}\right)dxdy\{\delta\},\end{aligned} \quad (1)$$

where

$$\begin{aligned}\left[\overline{N}\right] &= \left[N_1 \quad 0 \quad 0 \quad 0 \quad \vdots \quad \cdots\right], \\ \left[\overline{\overline{N}}\right] &= \left[0 \quad 0 \quad 0 \quad N_1 \quad \vdots \quad \cdots\right],\end{aligned} \quad (2)$$

and k is the distributed stiffness of the layer. So, the stiffness matrix takes the form:

$$[K]=\iint\left(\begin{matrix}[B]^T[D][B]+\left[\overline{B}\right]^T\left[\overline{D}\right]\left[\overline{B}\right]+\left[\overline{N}\right]^T k\left[\overline{N}\right]+\\ +\left[\overline{\overline{N}}\right]^T k\left[\overline{\overline{N}}\right]-\left[\overline{N}\right]^T k\left[\overline{\overline{N}}\right]-\left[\overline{\overline{N}}\right]^T k\left[\overline{N}\right]\end{matrix}\right)dxdy\,, \tag{3}$$

where

$$[D]=\frac{h^3}{12}\begin{bmatrix}\dfrac{E}{1-\nu^2} & \dfrac{E\nu}{1-\nu^2} & 0\\ \dfrac{E\nu}{1-\nu^2} & \dfrac{E}{1-\nu^2} & 0\\ 0 & 0 & \dfrac{E}{2(1+\nu)}\end{bmatrix},$$

$$\left[\overline{D}\right]=\frac{Eh}{2(1+\nu)k_s}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$[B]=\begin{bmatrix}0 & 0 & \dfrac{\partial N_1}{\partial x} & 0 & \vdots & \\ 0 & -\dfrac{\partial N_1}{\partial y} & 0 & 0 & \vdots & \cdots\\ 0 & -\dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_1}{\partial y} & 0 & \vdots & \end{bmatrix},$$

$$\left[\overline{B}\right]=\begin{bmatrix}\dfrac{\partial N_1}{\partial y} & -N_1 & 0 & 0 & \vdots & \cdots\\ \dfrac{\partial N_1}{\partial x} & 0 & N_1 & 0 & \vdots & \cdots\end{bmatrix} \tag{4}$$

and E is the modulus of elasticity, ν is the Poisson's ratio, h is the thickness of the plate, k_s is the shear correction factor assumed equal to 1.2.

The kinetic energy of the layer of the distributed masses is:

$$\begin{aligned}\mathrm{T}&=\frac{1}{2}\iint m\dot{w}^{*2}\,dxdy=\frac{1}{2}\iint m\left(\left[\overline{\overline{N}}\right]\{\dot{\delta}\}\right)^2 dxdy=\\ &=\frac{1}{2}\{\dot{\delta}\}^T\iint\left[\overline{\overline{N}}\right]^T m\left[\overline{\overline{N}}\right]dxdy\{\dot{\delta}\},\end{aligned} \tag{5}$$

where m is the distributed mass of the layer. So the mass matrix takes the form:

$$[M]=\iint\left(\begin{matrix}[N]^T\begin{bmatrix}\rho h & 0 & 0\\ 0 & \dfrac{\rho h^3}{12} & 0\\ 0 & 0 & \dfrac{\rho h^3}{12}\end{bmatrix}[N]+\\ +\left[\overline{\overline{N}}\right]^T m\left[\overline{\overline{N}}\right]\end{matrix}\right)dxdy\,, \tag{6}$$

where

$$[N]=\begin{bmatrix}N_1 & 0 & 0 & 0 & \vdots & \\ 0 & N_1 & 0 & 0 & \vdots & \cdots\\ 0 & 0 & N_1 & 0 & \vdots & \end{bmatrix} \tag{7}$$

and ρ is the density of the material of the plate.

Numerical investigation of the vibrations of a plate with the control layer of material

The analyzed object is a rectangular elastic plate with a layer of material with a fastened edge of the plate.

One of the powerful methods of visualization of dynamic results from FEM analysis is construction of interference fringe pattern corresponding to the time average holographic interferometry. It has many advantages over other vibration measurement techniques. It is a whole field non-invasive method and can be exceptionally useful if the amplitudes of the analyzed vibrations are in the range of micrometers. Though there exist a number of algorithms and techniques used for the interpretation of the measured holograms [6], the reconstruction of motion of structures from those holograms is a sensitive procedure in the sense of error accumulation.

The stroboscopic holographic image of the tenth eigenmode of the deflection of the layer of material is presented in Fig. 2.

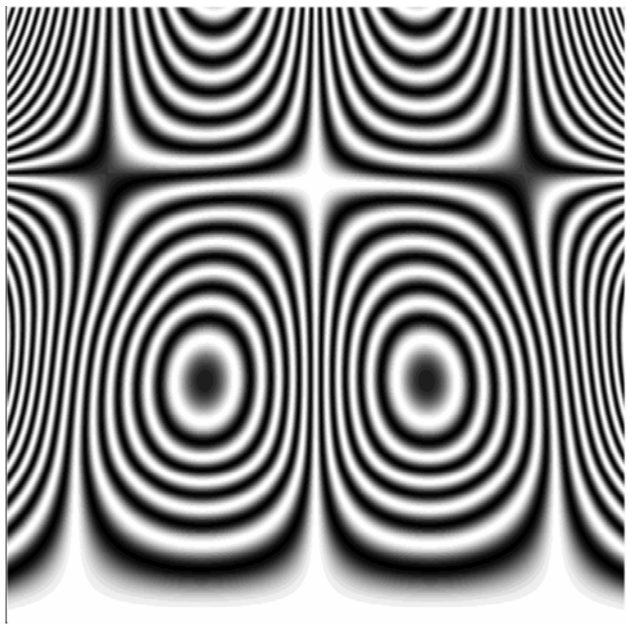

Fig. 2. **The stroboscopic holographic image of the tenth eigenmode of the deflection of the layer of material**

As mentioned earlier, visualization of holographic interference patterns has a number of drawbacks. Detection of variations from reference patterns requires the application of sophisticated algorithms [7], but even in that case the reconstructed patterns do not bring any information about the phase of the vibrations. Particularly, in vibration control applications, the information about the phase of vibrations is vital. Therefore, there exists a definite need for the development of new visualization techniques which could represent both the valuable information about the formation of fringes, both the phase of the occurring vibrations.

For such a representation of dynamic results the intensity mapping produced by holographic fringes construction algorithms is further on distorted by mapping represented in Fig. 3. Here *w* stands for the scalar deflection field of the plate; *I* – the intensity of the image; I_{max} – the maximum intensity associated with the image. This representation has the advantage over the usual representation by isolines or the holographic image because it enables to see the direction of change of the scalar variable.

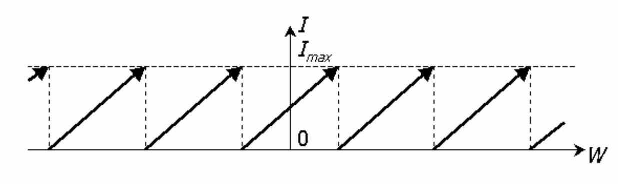

Fig. 3. **The secondary intensity mapping used in the representation of dynamic results**

The tenth eigenmode of the deflection of the layer of material represented by the intensity mapping is shown in Fig. 4.

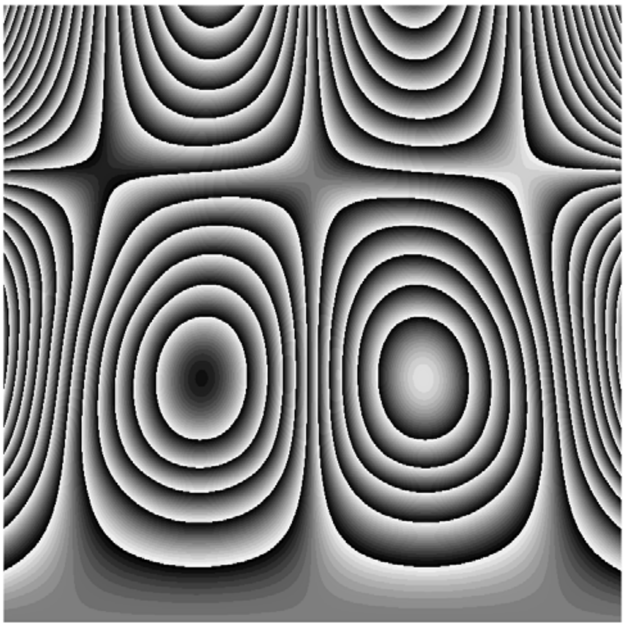

Fig. 4. **The tenth eigenmode of the deflection of the layer of material**

The regions of positive and negative deflections of the same eigenmode of the plate are shown in Fig. 5. It can be noted that the information about the phase of vibrations is preserved in Fig. 4.

Fig. 6 represents the vibrations of the control layer. This figure can be clearly interpreted from the point of fringe counting technique, and enable straightforward phase interpretation.

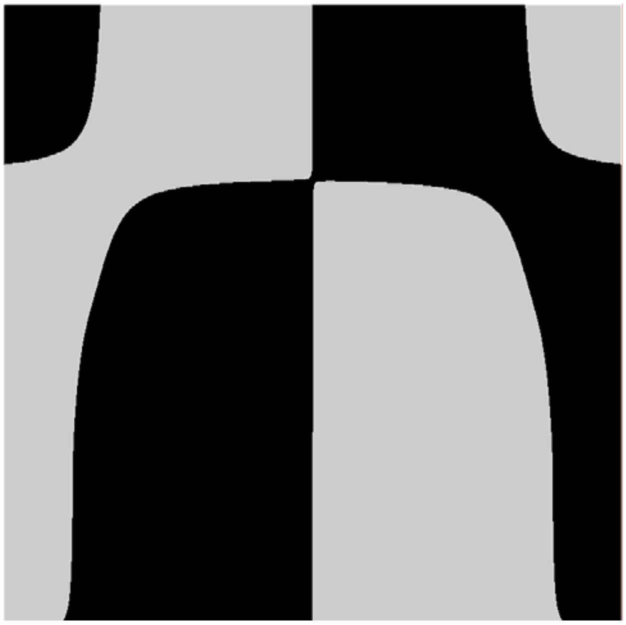

Fig. 5. **The regions of positive (grey) and negative (black) deflections of the tenth eigenmode of the plate**

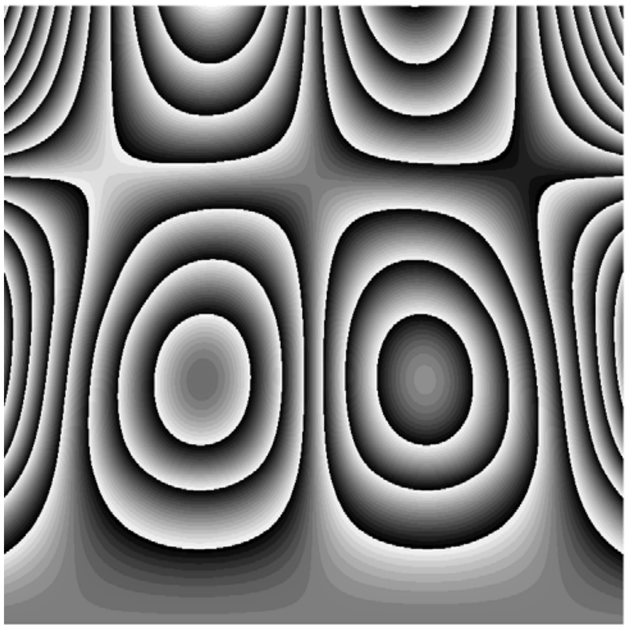

Fig. 6. **The tenth eigenmode of the deflection of the plate**

Conclusions

The new visualization technique is developed for the interpretation of dynamic FEM results. It has a number of advantages over existing techniques, particularly over the time average laser holography fringe pattern visualization. While preserving the useful properties of holograms it enables straightforward phase interpretation.

The developed technique is applied for the visualization and interpretation of the Winkler type active vibration control mechanism.

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M. Ragulskis, V. Kravčenkienė

Plokštelės su medžiagos sluoksniu osciliacinės dinamikos tyrimas

Reziumė

Sukurtas naujas dinaminių FEM rezultatų vizualizacijos metodas, turintis nemaža pranašumų, palyginti su kitais vizualizacijos metodais, taip pat ir su holografinių juostų formavimo metodais. Išlaikydamas naudingas holografinio metodo savybes, naujasis metodas leidžia tiesiogiai interpretuoti fazines reikšmes.

Šis metodas pritaikytas Vinklerio tipo vibracijų valdymo mechanizmo virpesiams vizualizuoti ir interpretuoti.

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