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The dynamics of a mechanical system excited parametrically by a travelling wave

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Notations

h	the non-dimensional coefficient of viscous friction
i, n	a natural member
y	the variable
α	the angle of excitation
ε	a dummy small parameter at the end of calculations assumed equal to 1
τ	the non-dimensional time
Ω	the non-dimensional angular velocity of rotational motion
ω	the frequency
–	over the member means the sign of overrating
~	over the member indicates that the constant part of this member with respect to the period is equal to 0

1. Introduction

In contemporary engineering systems with parametric excitation of vibrations and wave type are being used more videly [1– 6]. The systems with vibration excitation have been analysed more videly and some questions of their advancement have been solved. But until now the research of parametrically wave excited systems is in the initial stage. The purpose of this work is to analyse the system excited parametrically by a harmonic travelling wave, which is described by a simple linear and non-linear differential equation of the second order. The systems of this types are met in mechanical systems [3], [4], [6]. The results of research determining the dynamical characteristics of the system and revealing the specific qualities are useful for practice. This investigation is performed numerically and by an approximate analytical method.

2. Numerical analysis

Canonical form of the Mathieu equation, describing the motion of elliptic membrane, is given in [1]:

$$\frac{d^2y}{d\tau^2}+(a-2q\cos 2\tau)y=0 \qquad (2.1)$$

where the parameters $a, q \in R$, though the same requirement for τ is not necessary.

In [1] it is shown that the solution of Eq. 2.1 may take the form:

$$\mathrm{ce}_\nu(\tau,q)=\cos\nu\tau+\sum_{r=1}^{\infty}q^r c_r(\tau),$$

$$\mathrm{se}_\nu(\tau,q)=\sin\nu\tau+\sum_{r=1}^{\infty}q^r c_r(\tau),$$

$$a=\nu^2+\sum_{r=1}^{\infty}a_r q^r\ . \qquad (2.2)$$

Moreover, function pairs

$$\begin{cases}a_i(ce_i)\\ b_i(se_i)\end{cases},\ i=\overline{1,\infty} \qquad (2.3)$$

from the zones of unstable solutions of Eq. 2.1 in the parameter plane $a-q$. The function pairs touch the axis $q=0$ at the points $a=m$, where $m=1,2,3,...$

The stability zones of the dissipative Mathieu equation

$$\frac{d^2y}{d\tau^2}+h\frac{dy}{d\tau}+(a-2q\cos 2\tau)y=0 \qquad (2.4)$$

where h is the coefficient of linear viscous friction are numerically found and presented in Fig. 1. Due to presence of energy dissipation, the function pairs (2.3) do not touch the axis $q=0$.

Fig. 1. **The stability zones of Mathieu equation with dissipation at $h=0{,}1$. Black colour represents the stable zone, white – unstable. Horizontal axis denotes q, vertical *a*. The range $q\in[0{,}30]$, range of $a\in[0{,}30]$**

We introduce the modified Mathieu equation with zero derivative term holding a propagating unity:

$$\frac{d^2y}{d\tau^2}+h\frac{dy}{d\tau}+\left(a-2q\cos 2(\tau-ky)\right)y=0\,, \qquad (2.5)$$

where $k \in R$.

It is clear that presence of *k* modifies the structure of the solution. The stability zones at different values of *k* are presented in Fig. 2 – 5. All other parameters are analogous to Fig.1.

Fig. 2. **Stability zones at** $k = 0{,}01$

Important phenomena of Eq. 2.5 is that presence of even a small *k* dramatically changes the boundary line between the stable and unstable regions. Moreover, this line acquires fractal properties. Nevertheless, the global stability criterion is still valid for values of *q* sufficient larger than *a* – solution then turns unstable similarly as for Eq. 2.4.

Fig. 3. **Stability zones at** $k = 0{,}1$

Fig. 4. **Stability zones at** $k = 1$

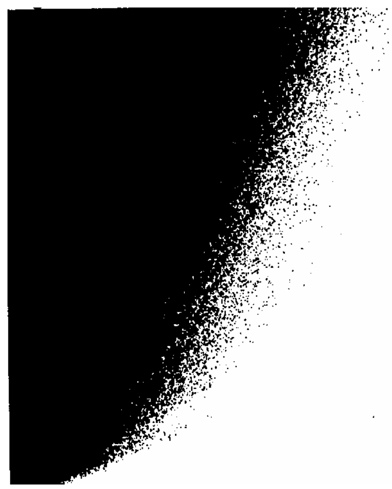

Fig. 5. **Stability zones at** $k = 100$

The practical analysis of a system loaded on a profile performing the oscillations of a propagating wave type, leads to a modification of Eq. 2.5:

$$\frac{d^2y}{d\tau^2}+h\frac{dy}{d\tau}+\left(a-2q\cos 2(\tau-ky)\right)\sin y=0\,. \qquad (2.6)$$

The stability analysis of this equation is in principle different – the derivative of solution $y(\tau)$ does not tend to infinity.

If the average turning velocity of the system in a steady state regime of motion is denoted as ω (the average of the derivative of solution), the solution of Eq. 2.6 may be classified into 3 types of reactions:

a) solution with $\omega = 0$;

b) solution with $\omega > 0$;

c) solution with $\omega > 0$.

If the numerical stability determination techniques are modified in a way, that stable solution is interpreted as a) case, and unstable – as b) and c) variants, the parametric plane would look like presented in Fig.6.

Fig. 6. **Classification of solutions at $k = 1, \alpha = 0$. Other parameters are analoque to those of Fig. 1– 5**

3. Analytical investigation

In those regions of space where chaos does not exist the steady state regimes may be analysed analytically.

3.1. The vibrating regimes

The steady state regime of Eq. 2.6 is sought in the form

$$y = \bar{y} + \tilde{y}, \tag{3.1}$$

where $\bar{y}$ and $\tilde{y}$ are the slow and fast regimes respectively [3]. The regime $\tilde{y}$ is described by the following differential equation

$$\frac{d^2\tilde{y}}{dt^2} + h\frac{d\tilde{y}}{dt} - 2q\sin(\bar{y}+\alpha)\cos 2(\tau - k\bar{y}) = 0. \tag{3.2}$$

The steady state fast motion on the basis of Eq. 3,2 is

$$\tilde{y} = \frac{q}{4+h^2}\sin(\bar{y}+\alpha)\cdot$$
$$\cdot\left[-2\cos 2(\tau - k\bar{y}) + h\sin 2(\tau - k\bar{y})\right]. \tag{3.3}$$

On the basis of Eq. (2.6)–(3.3) the slow motion is described by the following differential equation of motion

$$\frac{d^2\bar{y}}{dt^2} + h\frac{d\bar{y}}{dt} +$$
$$+\overline{\{a - 2q\cos 2[\tau - k(\bar{y}+\tilde{y})]\}\sin(\bar{y}+\tilde{y}+\alpha)} = 0 \tag{3.4}$$

or after performing the expansions into the power series with respect to $\tilde{y}$ it is obtained

$$\frac{d^2\bar{y}}{dt^2} + h\frac{d\bar{y}}{dt} +$$
$$+\overline{[a - 2q\cos 2(\tau - k\bar{y}) - 4qk\tilde{y}\sin 2(\tau - k\bar{y}) - \ldots]\cdot}$$
$$\overline{\cdot[\sin(\bar{y}+\alpha) + \tilde{y}\cos(\bar{y}+\alpha) + \ldots]} = 0. \tag{3.5}$$

By taking into account in the expansion of $\cos 2[\tau - k(\bar{y}+\tilde{y})]$ with respect to $\tilde{y}$ only the linear part and in the expansion of $\sin(\bar{y}+\tilde{y}+\alpha)$ the zero member of the power series with respect to $\tilde{y}$ it is obtained

$$\frac{d^2\bar{y}}{dt^2} + h\frac{d\bar{y}}{dt} + a\sin(\bar{y}+\alpha) -$$
$$-\frac{4hkq^2}{4+h^2}\sin^2(\bar{y}+\alpha) + \frac{2q^2}{4+h^2}\sin 2(\bar{y}+\alpha) = 0, \tag{3.6}$$

$$\frac{d^2\bar{y}}{dt^2} = 0, \frac{d\bar{y}}{dt} = 0. \tag{3.7}$$

Also assuming that *a* and *h* are small quantities

$$a \equiv \varepsilon a,$$
$$h = \varepsilon h, \tag{3.8}$$

from Eq. 3.6 it is obtained

$$\sin 2(\bar{y}+\alpha) +$$
$$+\varepsilon\left[\frac{(4+h^2)a}{2q^2}\sin(\bar{y}+\alpha) - 2hk\sin^2(\bar{y}+\alpha)\right] = 0. \tag{3.9}$$

From Eq. 3.9 the solutions $\bar{y}+\alpha$ are sought in the form of a power series with respect to ε

$$(\bar{y}+\alpha)_i = (\bar{y}+\alpha)_{i0} + \varepsilon(\bar{y}+\alpha)_{i1} + \varepsilon^2 \ldots \tag{3.10}$$

From the zero approximation it follows that

$$(\bar{y}+\alpha)_1 = n\pi \quad (n = 0,1,2\ldots) \tag{3.11}$$

are stable regimes, and

$$(\bar{y}+\alpha)_2 = \left(n + \frac{1}{2}\right)\pi \tag{3.12}$$

are unstable regimes.

From the Eq. (3.9) and (3.10) it is found

$$(\bar{y}+\alpha)_{i1} = \frac{4+h^2}{4q^2\cos 2(\bar{y}+\alpha)_{i0}}\cdot$$
$$\cdot\left[-a\sin(\bar{y}+\alpha)_{i0} + \frac{4q^2kh}{4+h^2}\sin^2(\bar{y}+\alpha)_{i0}\right]. \tag{3.13}$$

$(\bar{y}+\alpha)_{i1}$ determined by Eq. (3.13) give a qualitative answer in which direction the displacement of the average position takes place.

3.2. The rotational regimes

The solutions of Eq. (2.6) are sought in the form

$$y = \Omega\tau + \bar{y} + \tilde{y}. \tag{3.14}$$

Eq. 2.6 by taking into account Eq. 3.14 takes the form

$$\frac{d^2(\bar{y}+\tilde{y})}{dt^2} + h\left[\frac{d(\bar{y}+\tilde{y})}{dt} + \Omega\right]\cdot$$
$$\cdot\{a - 2q\{\cos 2[(1-k\Omega)\tau - k\bar{y}] - 2k\tilde{y}\sin 2[(1-k\Omega)\tau - k\bar{y}] + \ldots\}\}\cdot$$
$$\cdot[\sin(\Omega\tau + \bar{y}) + \Omega\tilde{y}\cos(\Omega\tau + \bar{y}) + \ldots] = 0. \tag{3.15}$$

In this case for the qualitative analysis it is sufficient to take into account only the zero member of the power series with respect to $\tilde{y}$ in Eq. 3.15. Thus from Eq.3.15 it is obtained

$$\frac{d^2\bar{y}}{dt^2}+h\left(\frac{d\bar{y}}{dt}+\Omega\right)-q\sin\left\{\left[\Omega-2(1-k\Omega)\right]\tau+\right.$$

$$\left.+(1+2k)\bar{y}\right\}-q\sin\left\{\left[\Omega+2(1+k\Omega)\right]\tau+(1-2k)\bar{y}\right\}=0\,. \quad (3.16)$$

In this case the rotational regimes may exist when

$$\Omega=\frac{2}{1+2k}, \quad (3.17)$$

the conditions of existence and stability of which are

$$\frac{2h}{(1+2k)q}<1, \quad (3.18)$$

$$\cos(1+2k)\bar{y}<0, \quad (3.19)$$

and the equation for the determination of $\bar{y}$ is

$$\sin(1+2k)\bar{y}=\frac{h\Omega}{q}. \quad (3.20)$$

Also the following rotational regime may exist

$$\Omega=-\frac{2}{1-2k}, \quad (3.21)$$

where the conditions of existence and stability are

$$\left|\frac{2h}{(1-2k)q}\right|<1, \quad (3.22)$$

$$(1-2k)\cos(1-2k)\bar{y}<0, \quad (3.23)$$

and the equation for the determination of $\bar{y}$ is

$$\sin(1-2k)\bar{y}=-\frac{2h}{(1-2k)q}. \quad (3.24)$$

Also other rotational regimes may exist, for the investigation of which when obtaining Eq. 3.16 it is necessary to use higher members of the power series with respect to ε in Eq. 3.15.

3. Conclusions

It is shown that in the mechanical system with parametric excitation by a harmonic travelling wave specific phenomena arise if compared with the simple harmonic excitation. Among those specific phenomena is chaos which exists in a wide region between the stable and unstable regimes in the separating zones. This depends much on the length and frequency of the travelling wave.

In the case of the wave excitation steady state regimes of rotational type also exist. Also between the vibrating and rotational regimes a wide separating zone is filled by the regimes of chaotic type.

In case of parametric wave excitation the vibrating and rotational regimes of deterministic type are analysed. The main parameters and the conditions of existence and stability of the simplest steady state regimes are determined.

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Bėgančiąja banga parametriškai žadinamos mechaninės sistemos dinamika

Reziumė

Analizuojama parametriškai žadinamos mechaninės sistemos dinamika. Parodyta, kad tuo atveju, kai sužadinimas yra bėgančiosios bangos tipo, ribos, skiriančios stabilius ir nestabilius judesio režimus, yra fraktalinės. Atlikta apytikslė analizinė stacionarių judesių analizė.

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