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Hybrid numerical – experimental holographic fluid interferometry: two-dimensional compressible fluid

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Introduction

Holographic interferometry enables effective analysis of flow problems and high frequency vibrations of fluids. Development of hybrid numerical – experimental fluid holographic methods is important for the interpretation of experimental results. In this paper the method of holographic interferometry is used for the analysis of the two-dimensional fluid vibrations. The eigenpairs of the fluid are determined by solving the problem using the finite element method. The analysis is based on [1, 2, 3, 4].

Here the plotting procedure is developed including the smoothing of the values of volumetric strain in order to obtain more realistic holographic images of the eigenmodes of a fluid. Conventional FEM analysis techniques are based on the approximation of nodal displacements (not the volumetric strains) via the shape functions [1, 2, 3]. Conventional FEM would require unacceptably dense meshing for producing sufficiently smooth images. Therefore the technique for smoothing of the generated images representing the distribution of the volumetric strains and calculated from the displacement distribution is developed. The smoothing technique is similar to conjugate approximation used for the calculation of nodal values of stresses in [3] and enables to obtain the images of better quality on a coarse mesh by using the displacement formulation for the calculation of the eigenmodes.

Numerical model of the system

The mass matrix of the fluid is:

$$M = \int N^T \rho N dxdy, \tag{1}$$

where ρ is the density of the fluid, N is the matrix of the shape functions defined from:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = N\delta, \tag{2}$$

where u, v are the displacements in the directions of the axes of coordinates x and y, δ is the displacement vector, that is:

$$N = \begin{bmatrix} N_1 & 0 & \cdots \\ 0 & N_1 & \cdots \end{bmatrix}, \tag{3}$$

where N_i are the shape functions.

The stiffness matrix of the fluid is:

$$K = \int \left(\overline{B}^T \rho c^2 \overline{B} + \widetilde{B}^T \lambda \widetilde{B} \right) dxdy + \int \overline{N}^T \rho g \overline{N} dx, \tag{4}$$

where c is the speed of sound, λ is the penalty parameter for the condition of irrotationality, g is the acceleration of gravity, the second integral is over the free surface only, the matrix $\overline{B}$ is defined from:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \overline{B}\delta, \tag{5}$$

that is:

$$\overline{B} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \cdots \end{bmatrix}, \tag{6}$$

the matrix $\widetilde{B}$ is defined from:

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \widetilde{B}\delta, \tag{7}$$

that is:

$$\widetilde{B} = \begin{bmatrix} \frac{\partial N_1}{\partial y} & -\frac{\partial N_1}{\partial x} & \cdots \end{bmatrix}, \tag{8}$$

the matrix $\overline{N}$ of the shape functions is defined from:

$$v = \overline{N}\delta, \tag{9}$$

that is:

$$\overline{N} = [0 \quad N_1 \quad ...]. \tag{10}$$

Construction of the holographic image

The phase of the light from the laser beam is given by [4]:

$$\Psi(x, y) = \frac{2\pi}{\lambda} \left[n_0 - n_{flow}(x, y) \right] h, \tag{11}$$

where h is the distance that the light travels through the fluid, λ is the wavelength of the laser beam, n_0 and n_{flow} are the refractive indexes in the initial and flow conditions respectively.

The refractive index is expressed as [4]:

$$n(x, y) = 1 + \beta \frac{\rho(x, y)}{\rho_0}, \tag{12}$$

where ρ_0 is the density constant in the region of the flow in the status of equilibrium, β is the constant of proportionality.

From the previous relationships it follows that:

$$\Psi(x, y) = \Psi_0 - k\rho_{flow}(x, y), \tag{13}$$

where the initial phase ψ_0 and the coefficient of proportionality k are expressed like:

$$\Psi_0 = \frac{2\pi}{\lambda} \beta h, \tag{14}$$

$$k = \frac{2\pi}{\lambda}\frac{\beta}{\rho_0}h. \quad (15)$$

Further it is assumed that:

$$\tilde{\rho}(x,y,t) = \rho_{flow}(x,y,t) - \rho_0, \quad (16)$$

where the deviation of the density from the density in the status of equilibrium is small:

$$|\tilde{\rho}(x,y,t)| << \rho_0. \quad (17)$$

Then:

$$\Psi(x,y) = \overline{\Psi_0} - k\tilde{\rho}(x,y,t), \quad (18)$$

where

$$\overline{\Psi_0} = \Psi_0 - k\rho_0. \quad (19)$$

Further it is assumed that the density and the displacements are harmonically varying in time:

$$\rho_{flow}(x,y,t) = \rho_0 + \tilde{\rho}^*(x,y)\cos(\omega t), \quad (20)$$

and

$$u(x,y,t) = \tilde{u}^*(x,y)\sin(\omega t), \quad v(x,y,t) = \tilde{v}^*(x,y)\sin(\omega t), \quad (21)$$

where ω can coincide with the frequency of oscillations of the appropriate eigenmode.

The equation of continuity of the fluid can be represented as:

$$\frac{\partial\rho}{\partial t} = -\rho_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \quad (22)$$

Eq. (22) together with the Eq. (20) and (21) gives:

$$\tilde{\rho}^*(x,y)\omega = \rho_0\left(\frac{\partial\tilde{u}^*(x,y)}{\partial x} + \frac{\partial\tilde{v}^*(x,y)}{\partial y}\right). \quad (23)$$

So after performing the initial calibration of the phase of the laser beam its intensity I for the stroboscopic image can be expressed as:

$$I(x,y) = \cos^2\left(a\left(\frac{\partial\tilde{u}^*(x,y)}{\partial x} + \frac{\partial\tilde{v}^*(x,y)}{\partial y}\right)\right), \quad (24)$$

where the coefficient a can be expressed from the equations (18) and (23):

$$a = -\frac{k\rho_0}{\omega}. \quad (25)$$

It is assumed that the fluid performs high frequency vibrations according to the eigenmode (the frequency of excitation is about equal to the eigenfrequency of the corresponding eigenmode and the eigenmodes are not multiple). The vibrations of the system are registered stroboscopically when the structure is in the state of extreme deflections according to the eigenmode. In this case the problem is to obtain the volumetric strains present in Eq. (24) of acceptable quality for the eigenmode (the eigenmode of volumetric strains).

The volumetric strains at the points of numerical integration of the finite element are calculated in the usual way:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = [B]\{\delta_0\}, \quad (26)$$

where $\{\delta_0\}$ is the vector of nodal displacements of the eigenmode; $[B]$ is the matrix relating the volumetric strains with the displacements:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \cdots \end{bmatrix}. \quad (27)$$

The displacements are continuous at interelement boundaries, but the calculated volumetric strains due to the operation of differentiation are discontinuous. The eigenmode of strains is obtained by minimising the following error:

$$\frac{1}{2}\iint\left(\begin{gathered}([N]\{\delta_V\} - (u_x + v_y))^2 + \\ + \lambda\left(\left(\frac{\partial(u_x + v_y)}{\partial x}\right)^2 + \left(\frac{\partial(u_x + v_y)}{\partial y}\right)^2\right)\end{gathered}\right)dxdy = $$
$$= \frac{1}{2}\iint\left(\begin{gathered}([N]\{\delta_v\} - (u_x + v_y))^2 + \\ + \lambda\{\delta_v\}^T[B^*]^T[B^*]\{\delta_v\}\end{gathered}\right)dxdy, \quad (28)$$

where λ - the smoothing parameter; $\{\delta_v\}$ - the vector of nodal values of $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ (the eigenmode of volumetric strains); $[N]$ - the row of the shape functions of the finite element; $[B^*]$ - the matrix of the derivatives of the shape functions:

$$[N] = [N_1 \quad N_2 \quad \cdots \quad N_n], \quad [B^*] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_n}{\partial y} \end{bmatrix}. \quad (29)$$

This leads to the following system of linear algebraic equations for determination of the volumetric strains:

$$\iint\left([N]^T[N] + [B^*]^T\lambda[B^*]\right)dxdy \cdot \{\delta_v\} = \iint[N]^T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dxdy. \quad (30)$$

The choice of the smoothing parameter is performed interactively from the qualitative view of the digital holographic images. When the parameter is too small the images are insufficiently smooth because of the volumetric strains calculated from the displacement formulation. When the parameter is too large an oversmoothed image is obtained which may look acceptable, but be far from a real holographic image.

Numerical investigation

The rectangular domain is analysed. The lower boundary is a rigid boundary and the displacements normal to it are set to zero. The upper surface is assumed to be a free surface. The periodic boundary conditions in the x direction are assumed: that is the values of the corresponding displacements on the left and the right boundaries for the same values of the y coordinate are assumed to be mutually equal.

The eigenmodes are calculated and on their basis the stroboscopic holographic images are constructed. The obtained unsmoothed image for the second eigenmode is shown in Fig. 1. The smoothed images for the second and the third eigenmodes are shown in Fig. 2 and Fig. 3.

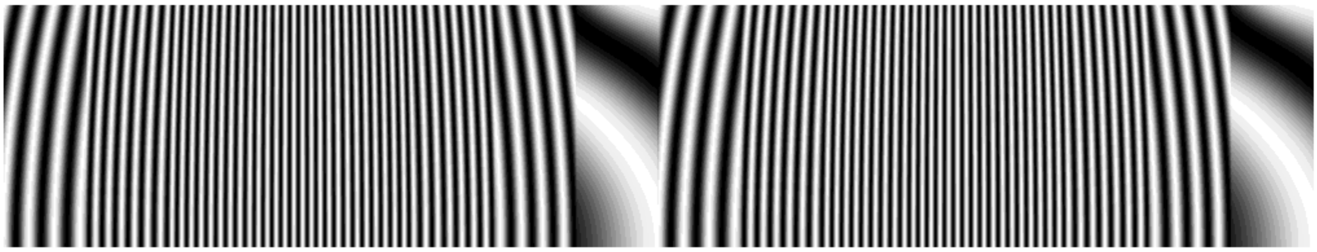

Fig. 1. **The unsmoothed stroboscopic holographic image for the second eigenmode**

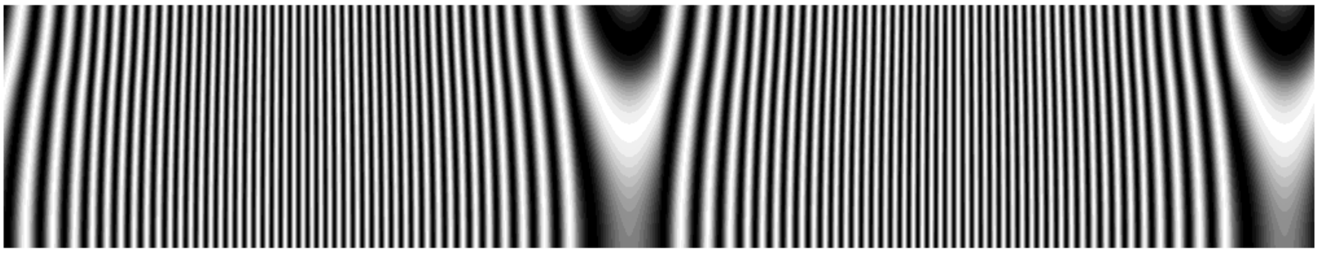

Fig. 2. **The smoothed stroboscopic holographic image for the second eigenmode**

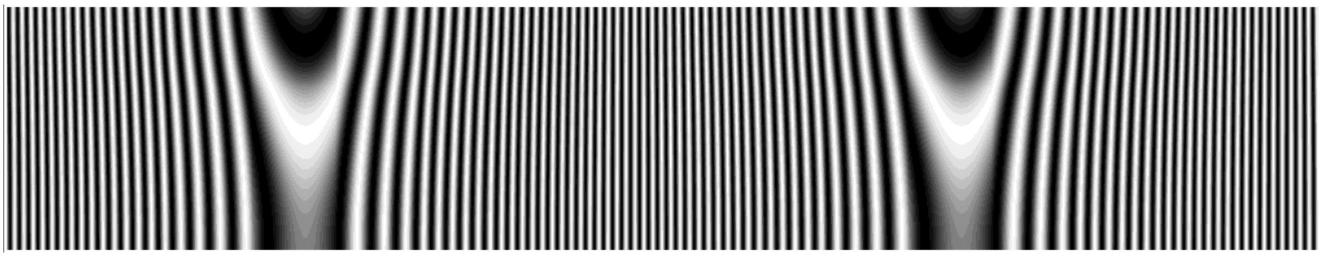

Fig. 3. **The smoothed stroboscopic holographic image for the third eigenmode**

Conclusions

The method of holographic interferometry is applied for the two-dimensional problem of vibrations according to the eigenmode by using the stroboscopic method lightening the structure in the state of extreme deflections.

The obtained stroboscopic holographic images of the multiple eigenmodes for the analysed periodic system may be effectively used for the excitation of wave motion in the fluid transport systems. As the displacement based FEM formulation is used for volumetric strains based holographic analysis, the introduced smoothing procedure enables generation of holographic images on coarse finite element meshes.

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Hibridinė skaitmeninė-eksperimentinė skysčio holografinė interferometrija: dvimatis spaudžiamas skystis

Reziumė

Naudojant poslinkių formuluotę ir tūrinės deformacijos skaičiavimą glotninant, gautos skysčio savos formos virpesių stroboskopinės holografinės interferogramos. Analizuoto uždavinio kartotinės formos periodinėje konstrukcijoje taikomos banginiam transportavimui.

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