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Transmission of ultrasonic waves through fluid-loaded composite multilayered anisotropic structures

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Introduction

Composite materials have been subject of permanent interest during last few decades. Composites offer many advantages in comparison to alloys, for example, high strength and stiffness, excellent fatigue properties and corrosion resistance. On other hand, they have several disadvantages like low fracture toughness and moisture absorption [1]. In addition to that, inspections during exploitation may be difficult and maintenance complicated or impossible. For non-destructive testing and evaluation ultrasonic methods are widely used. For this purpose immersion or dry coupling ultrasonic techniques are exploited. They enable to find discontinuities like delaminations, to estimate quality of bonds, to determine elastic properties and even the morphology of the material under a test.

The problems related to the interaction of elastic waves with fluid-loaded multilayered solids have been widely studied [2, 3]. In most cases, layered structures consist of two or more layers bonded to each other in some order. For analysis of wave propagation in such media, solutions are obtained in terms of the displacements and stresses of each component. By satisfying appropriate boundary conditions, characteristic equations are constructed which involve the properties of all layers.

This paper presents results of investigation of ultrasonic wave transmission through multi-layered composite plates immersed in water. The experiments were carried out on bi-axially laminated GLARE® specimen at various frequencies and angles of incidence of the ultrasonic wave. The data are compared to the results of a theoretical analysis based on exact analytical treatment of wave propagation in a fluid-loaded anisotropic plate in conjunction with the transfer matrix approach.

The transfer matrix technique was introduced originally by Thomson [4] and later refined by Haskell [5] and have been used in geophysics and ocean acoustic. Most of available literature on layered media is restricted to the study of situations where the individual material layer is isotropic. Nayfeh [6] and Chimenti [7] developed the transfer matrix method for ultrasonic propagation in generally anisotropic solids.

In this work we adapted this method for analysis of multilayered composite orthotropic media. That enabled to derive exact analytical expressions for the transmission and reflection coefficients of ultrasonic wave. The results obtained may be useful for non-destructive characterization of multi-layered composite materials used for aerospace applications.

Application of the transfer matrix approach for anisotropic composite layered media

Let us consider a plate consisting of an arbitrary number n of anisotropic layers, each exhibiting as low as orthotropic symmetry, rigidly bonded at their interfaces and stacked normal to the x_3 axis of the global orthogonal Cartesian system $x_i=(x_1, x_2, x_3)$. The interfaces are parallel to the x_1-x_2 plane, which is also chosen to coincide with the bottom surface of the layered structure. To maintain generality we shall assume each layer to be arbitrarily oriented in the x_1-x_2 plane. To describe the relative orientation of the layers, we assign an index k for each layer, k=1, 2,…n, and such a local Cartesian coordinate system $(x'_i)_k$ that its origin is located in the middle plane of the layer with the axis $(x'_3)_k$ normal to it.

Thus the layer k extends from $-d^{(k)}/2$ to $d^{(k)}/2$, where $d^{(k)}$ is the thickness of the layer and d equals to the total thickness of the plate. The material orientation in the k-th layer is given by a rotation angle ϕ_k between $(x'_1)_k$ and x_1.

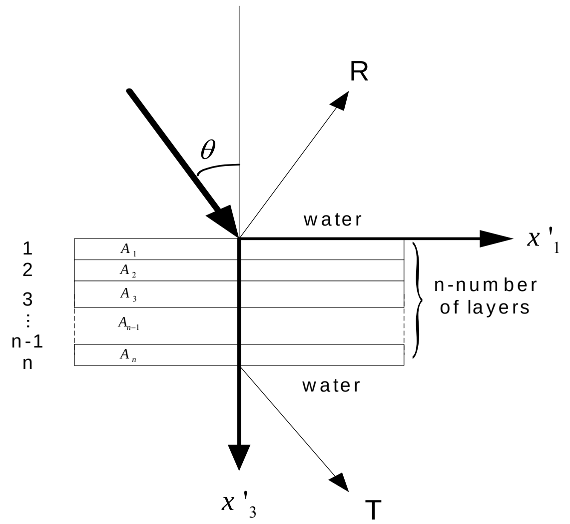

Fig. 1. **Model geometry**

We assume that a plane wave is incident to the upper layer at an arbitrary angle in the x_1-x_3 plane. Our task is to find the ultrasonic field transmitted through the layered structure. Therefore the analysis is carried out in the coordinate system formed by incident and reflected planes rather than by material symmetry axes [3]. Accordingly, the primed system $(x'_i)_k$ is rotated with one set of material symmetry axes while the global unprimed system x_i

remains invariant. This approach leads to significant simplification in our algebraic analysis and computation.

In this section we shall find transfer matrixes for each layer. This matrix relates the displacements and stresses of one face to those of the other of the k -th layer with respect to the primed coordinate system. Elastic field equations of the k -th layer are given by the momentum equation:

$$\frac{\partial \sigma'_{ij}}{\partial x'_j} = \rho' \frac{\partial^2 u'_i}{\partial t^2}, \quad (1)$$

and from the general constitutive relations for anisotropic media:

$$\sigma'_{ij} = c'_{ijkl}\, e'_{kl}, \quad (2)$$

where u_i' and σ_{ij}' are the displacements and the stresses, c_{ijkl}' are the elastic constants of the k -th layer and e_{kl}' is the strain.

The formal solutions for the each layer can be written as:

$$\begin{bmatrix} u_1 \\ u_3 \\ \sigma^\bullet_{33} \\ \sigma^\bullet_{13} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ W_1 & W_2 & W_3 & W_4 \\ D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \end{bmatrix} \begin{bmatrix} U_{11}E_1 \\ U_{12}E_2 \\ U_{13}E_3 \\ U_{14}E_4 \end{bmatrix}, \quad (3)$$

where u_1, u_3 and $\overset{*}{\sigma}_{33}, \overset{*}{\sigma}_{13}$ are the displacements and the stresses in the x_1, x_3 directions, U_{1q} are the displacement amplitudes, E_q is the attenuation. The variables W_q, D_{1q}, D_{2q} are given in appendix.

Eq.3 can be used to relate the displacements and stresses at $(x'_i)_k = -d^{(k)}/2$ to those at the opposite surface $(x'_i)_k = d^{(k)}/2$. This is done by the eliminating the common amplitudes $U_{11},\ldots,U_{14}$ and getting:

$$P_k^+ = A_k P_k^-, \quad (4)$$

where

$$P_k^{\pm} = \{[u_1, u_3, \overset{*}{\sigma}_{33}, \overset{*}{\sigma}_{13}]^T_{\pm}\}_k, \quad (5)$$

defines the variables column specialized to the upper and lower surfaces of the k -th layer and

$$A_k = X_k D_k X_k^{-1}, \quad (6)$$

where X_k is the (4x4) square matrix of Eq.3 and D_k is a (4x4) matrix which entries are E_q .

The matrix A_k constitutes the transfer matrix for the orthotropic k -th layer. It is valid for the wave incident on the layer at an arbitrary angle θ from the normal x_3 and at any azimuthal angle ϕ . By applying the above described procedure for each layer followed by invoking the continuity of the displacement and stress components at the layer interfaces, we can finally relate the displacements and stresses at the upper plane of the layered plate to those at its bottom, via multiplication of transfer matrixes.

Fluid boundary

The plate is completely immersed in a fluid (water). The ultrasonic wave is assumed to be periodic and originating in the fluid and incident on the plate at an arbitrary angle. The displacements and stresses within the upper and bottom parts of the fluid are given properly specializing and recognizing absence of shear deformations within the fluid. For the upper fluid region:

$$\begin{bmatrix} u_1 \\ u_3 \\ \sigma^\bullet_{33} \end{bmatrix}^U = \begin{bmatrix} 1 & 1 \\ \alpha_f & -\alpha_f \\ i\xi\rho_f c^2 & i\xi\rho_f c^2 \end{bmatrix} \begin{bmatrix} e^{i\xi\alpha_f (x'_3 + \frac{d}{2})} \\ R \cdot e^{-i\xi\alpha_f (x'_3 + \frac{d}{2})} \end{bmatrix}, \quad (8)$$

and for the bottom fluid region:

$$\begin{bmatrix} u_1 \\ u_3 \\ \sigma^\bullet_{33} \end{bmatrix}^L = \begin{bmatrix} 1 \\ \alpha_f \\ i\xi\rho_f c^2 \end{bmatrix} \begin{bmatrix} T \cdot e^{i\xi\alpha_f (x'_3 + \frac{d}{2})} \end{bmatrix}, \quad (9)$$

where $\alpha_f^2 = \left(c^2 / c_f^2 \right) - 1$, c is the common phase velocity. The common phase velocity is equal to $c_k / \sin(\theta_k)$, where c_k and $\sin(\theta_k)$ are the velocity and the propagation angle in k-th layer , ρ_w is the water density, ξ is the x_1-component of the wave number.

Combination of equations yields the following expression for the transmission coefficient of ultrasonic wave through the plate:

$$T(f, \theta) = \frac{\det[A^T]}{\det[A]}. \quad (11)$$

Experimental technique

Experimental investigation of ultrasonic wave transmission through fluid-coupled plates was performed using two different specimens – aluminum and multilayered GLARE composite plates.

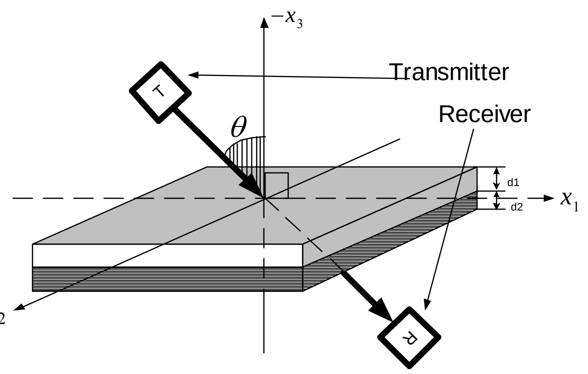

Fig. 2. **Experimental setup**

The experimental equipment as shown in Fig. 5 includes an ultrasonic generator, a digitizing oscilloscope, two piezoelectric transducers, signal amplifiers and a personal computer. The system is configured so that the angle of the sample between the two transducers can be adjusted precisely by special electromechanical positioning device (Table 1). The signals are acquired at each angle of transmission through the sample. An additional signal is acquired without the sample as a reference signal to characterize the propagation through water. The transmission coefficients for each sample at different angles may be found by the deconvolution of the reference signal from the received signal:

$$T(f)=\left|\frac{fft(S_{in})}{fft(S_{ref})}\right|, \qquad (12)$$

where $S(j\omega)$ is the frequency spectrum of the signal, fft denotes the fast Fourier transform:

$$S(j\omega)=\sum_{n=-\infty}^{\infty} x_{in}(n)e^{-j\omega n} . \qquad (13)$$

The experiments were performed with the purpose of verification of the model and also to investigate the sample material properties.

For transmission and reception focused piezoelectric transducers with a nominal center frequency of 5 MHz and a diameter 12mm were used. The distance between transducers was selected 30 cm what corresponds to the far zone and the sample was located in the middle between the transducers. The signals were acquisitted and digitized by a digital storage oscilloscope (HP54645) and transferred to a personal computer using a IEE488 interface.

The orientation angle of the sample between the transducers can vary in the interval [0, 360°] by a scanning step 0.015° and repeatability ±0.013°.

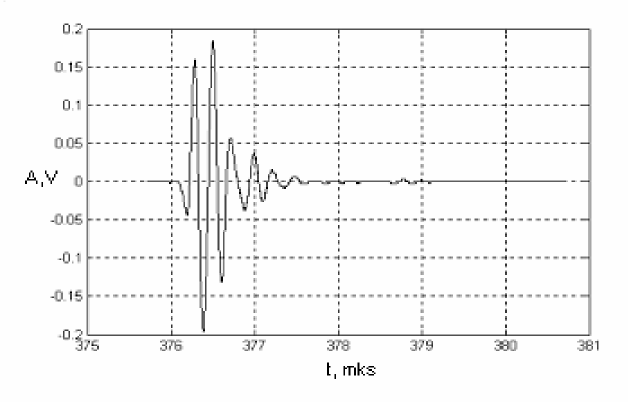

Fig. 3. **Experimentally measured waveforms of the reference signal**

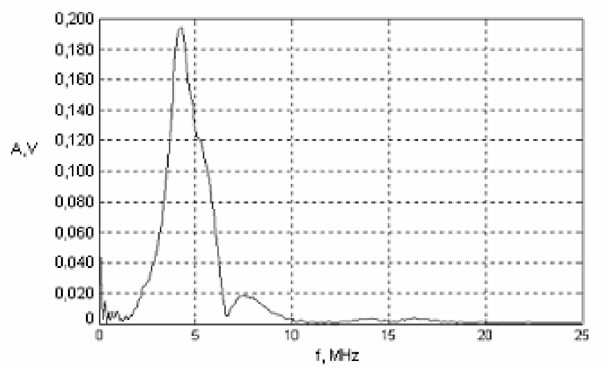

Fig. 4**. Frequency spectrum of the reference signal**

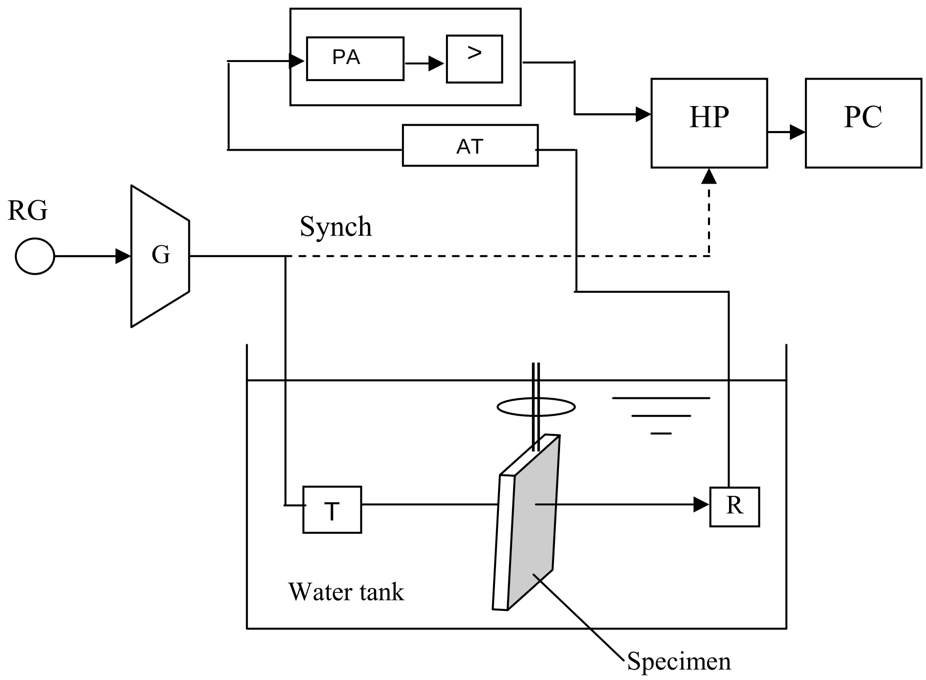

Fig. 5. **Experimental system configuration: G – arbitrary waveform generator, AS-output stage generator, T,R – transmitting and receiving ultrasonic transducers, AT – attenuator, PA – preamplifier, HP – digital oscilloscope, PC – personal computer**

Table 1. **Data of Electromechanical positioning system**

Coordinate	Max. value [mm]	Alteration interval [µm/step]	Reiteration [µm]
X	1000	9	±50
Y	600	9	±50
Z	360	1	±10

For experimental investigation we have chosen two plates – aluminum and GLARE 3/2 composite samples. Aluminum represents one of the high-symmetry materials and possesses the same elastic properties in different directions. Thickness of aluminum plate was 1mm. The GLARE 3/2 represents low-symmetry multilayered composite structure now widely used for aerospace applications. This structure consists of three Al and two glass fiber/epoxy layers. This material has different elastic properties in-plane and off-plane directions. Data of the GLARE 3/2 sample are presented in Tables 2 and 3.

Table 2. **GLARE specification**

Laminate number	7103A
Material configuration	GLARE 3 3/2 0,3
Type of composite	FM 94
Type of metal	2024 T3

Table 3. **Data of each layer**

Layer	Thickness (mm)	Orientation (°)
Al	0,3	0
Composite	0,125	0
Composite	0,125	90
Al	0,3	0
Composite	0,125	0
Composite	0,125	90
Al	0,3	0

Numerical simulation and experimental verification

The simulated and experimentally measured frequency-dependent transmission coefficients at various incident angles of ultrasonic wave are shown in Fig. 6-11. At some incidence angles (Fig. 10) and frequencies abrupt discontinuities exist in the simulated transfer response. They are caused by numerical instabilities in the model [7,8]. The effect of these instabilities is reduced with an increase in the number of the frequencies used. The instabilities in the model are removed by averaging.

A good agreement between measured and simulated transfer responses was found in the range of the incident angles from 0° to 15°. In the range 15°-70° the simulation results were rather similar to the experimentally measured. At bigger incident angles (70°**-**90°**)** the significant instabilities in the model appeared and no results were obtained.

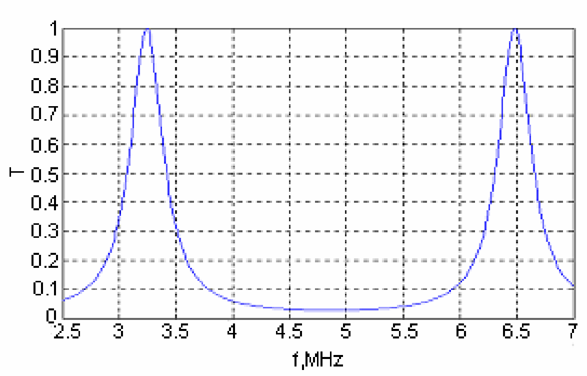

Fig.6. **Simulated transmission response of aluminum plate. Incidence angle of 5°**

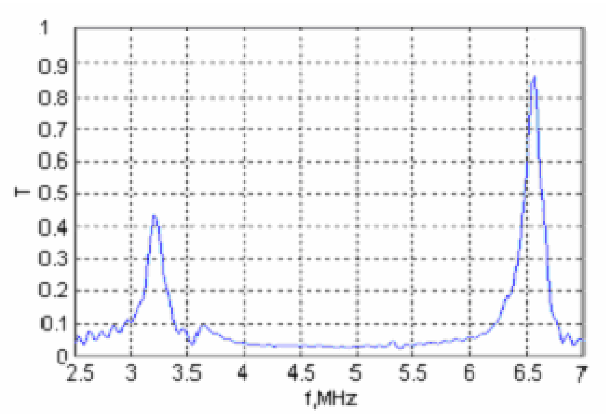

Fig. 7. **Measured transmission response of aluminum plate. Incidence angle of 5°**

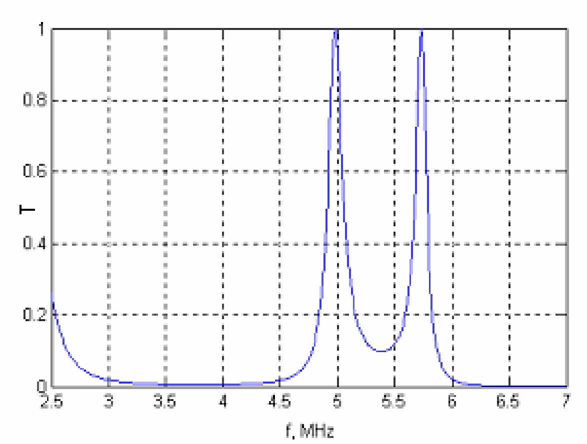

Fig.8. **Simulated transmission response of the multilayered GLARE 3/2 plate. Incidence angle 5°**

Conclusions

Theoretical analysis and results of experimental investigation of interaction of ultrasonic waves with multilayered orthotropic structures are presented. It is supposed that the structure is immersed into water and the plane wave propagates at an arbitrary angle with respect to the plate consisting of an arbitrary number of different material layers. The transmission coefficients of an ultrasonic wave through the structure were obtained, which enable to use them non-destructive estimation of the multi-layered samples.

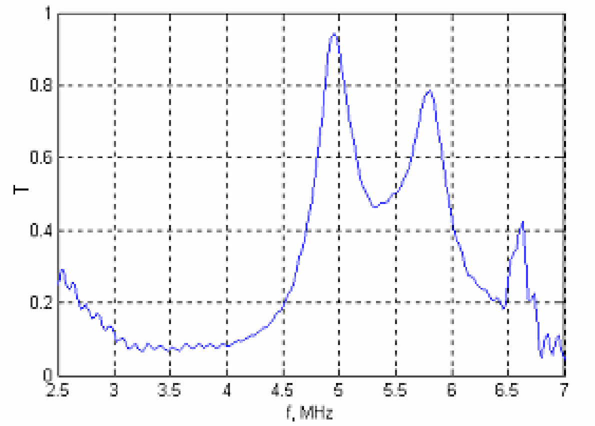

Fig.9. **Measured transmission response of the multilayered GLARE 3/2 plate. Incidence angle 5°**

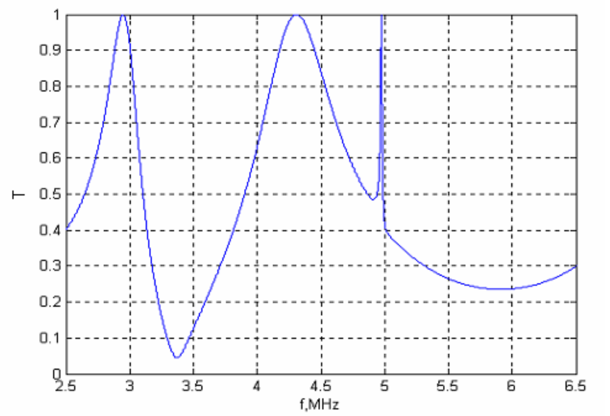

Fig.10. **Simulated transmission response of the multilayered GLARE 3/2 plate. Incidence angle 35°**

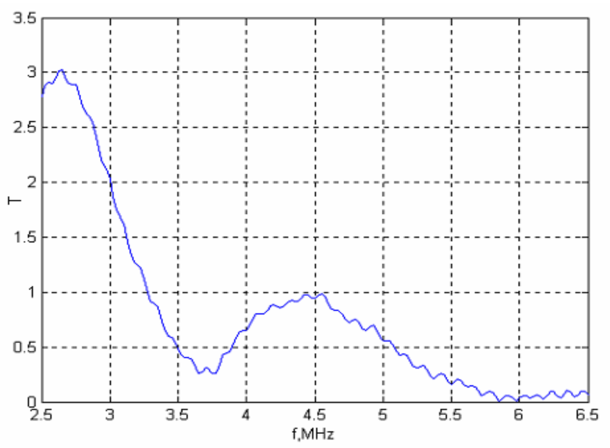

Fig.11. **Measured transmission response of the multilayered GLARE 3/2 plate. Incidence angle 5°**

Measurements using one-layer aluminum plate and biaxialy laminated GLARE 3/2® specimens totally immersed in water were performed. The measurement results were compared to the results of the theoretical analysis based on the transfer matrix approach. Comparison have shown that correspondence between the simulated and experimental data is reasonably good what confirms accuracy and efficiency of the developed numerical model.

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Appendix

$$W_q = \frac{K_{11}(\alpha_q)}{K_{13}(\alpha_q)} = \frac{\rho c^2 + C_{11} - C_{55}\alpha_q^2}{(C_{13} + C_{55})\alpha_q}$$

$$D_{1q} = C_{13} \sin^2\theta + C_{55}\alpha_q W_q$$

$$D_{3q} = C_{55}(\alpha_q + W_q \sin^2\theta)$$

$$E_q = e^{i\xi\alpha_q x_3'}$$

where ξ is the x_1 component of the wave number, c is the common phase velocity ($c=\omega/\xi$) along x_1, ω is the circular frequency, α is an unknown ratio of the wave number components along the x_1 and x_3 directions, defined by solving linear equation system and U_j is the displacement amplitude.

G. Butėnas, R. Kažys

Ultragarsinių bangų perdavimas per skysčiu apkrautas kompozicines daugiasluoksnes anizotropines struktūras

Reziumė

Daugiasluoksnes kompozicines struktūras pradedama plačiai naudoti aviacijoje. Straipsnyje teoriškai bei eksperimentiškai ištirtas ultragarsinių bangų perėjimas per izotropinę metalinę ir anizotropinę daugiasluoksnę kompozicinę plokštes vandenyje, esant įvairiems bangos kritimo kampams. Teorinis tyrimas atliktas perdavimo matricų metodu. Parodyta, kad teoriniai skaičiavimai gerai atitinka eksperimentinius rezultatus. Šie rezultatai gali būti panaudoti kuriant naujus daugiasluoksnių kompozitų ultragarsinius diagnostikos metodus.

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