Analysis of the steady state operation conditions of wave vibrodrives

G. Baurienė[1], L. Patašienė[1], K. Pilkauskas[2]

[1]*Faculty of Mechanical Engineering, Kaunas University of Technology,*
Kęstučio 27. LT-44025, E-mail: genovaite.bauriene@ktu.lt, laima.patasiene@ktu.lt
[2]*Faculty of Mechanical Engineering, Kaunas University of Technology,*
Mickeviciaus 37, LT-44244, E-mail: kestutis.pilkauskas@ktu.lt

Introduction

The operational principle of wave vibrodrive is based on friction interaction of an input link in which a high frequency traveling wave oscillation are excited with an output link [1, 2, 3]. The character of motion of the output link is determined by the vibration character of the input link, external resistance forces, the shape of contacting surfaces, etc.

The results of analytical investigation of a wave vibrodrive with a ring shape exciter which contacts the output link by a point in case of kinematic excitation are presented.

Vibrations of mechanical system in which a dry friction is present are described by nonlinear differential equations. The feature of motion of such systems is that due to nonlinear influence of a dry friction, the motion of the links can be separated into different phases. The motion of the phases is described by different easily integrated differential equations. The differential equations describing the motion of nonlinear systems are mainly used for the determination of transition instant from one phase to another.

Taking this into account, the dynamical model of the vibrodrive can be represented by a simplified nonlinear (quasilinear) mechanical system and the differential equations describing its motion can be solved applying so called method of junction [4, 5].

The essence of this method is that a nonlinear mechanical system in separate phases of its motion is analysed as linear.

For the solutions of each phase such constants of integration are selected that the solutions of adjacent phases would transit to each other fluently. In general solution the constants of the first phase are determined from the initial conditions of the process. The end values of the first phase coordinates and velocities are used as initial conditions of the second phase and so on. The calculation is performed till the steady state is obtained or it is approached with necessary accuracy.

The feature of the steady state is that in each phase there are two instants in the time domain when the velocity becomes equal zero. If in time the steady motion approaches some limit motion, the later with a great probability can be considered as a asymptotically stable motion.

Theoretical investigation of the steady state motion

Wave vibrodrive is described by a simplified dynamical model which consists of an absolutely rigid particle 1 moving according the defined law and, output link 2 which is pressed to the input link 1 and which can perform motion (see Fig.1) in one direction. The case under investigation is when the output link 2 performs translation.

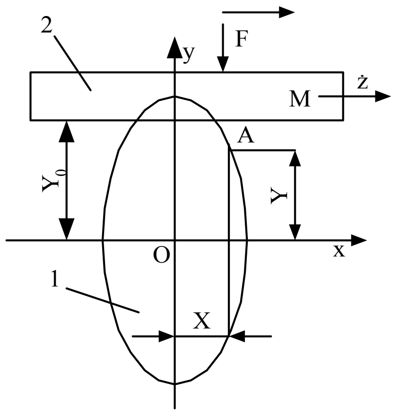

Fig.1. **Kinematic scheme of wave vibrodrive: 1 – input link; 2 – output link**

The input and output links interact in the place of their contact by the forces of sliding friction. The input link 1 moves by an elliptic trajectory, which is described by the following equations:

$$X = A\sin(\omega t + \psi_0),$$
$$Y = B\cos\omega t, \qquad (1)$$

here X, Y coordinates of the ellipsis, A, B amplitudes of the vibrations, ω – angular frequency of the vibrations, ψ_0 – phase shift (in order to compare analytical investigations it is assumed to be zero).

When determining the forces of sliding friction only the dry friction force appearing at the contact point of the input and output links is taken into account.

The force of the dry friction F_1 equals $F_1 = f_0 \, N \, \mathrm{sign}(\dot{Z} - \dot{X})$, here Z is the coordinate of the output link 2.

Normal reaction $N = M\ddot{Y} + F$, here F is the pressing force of the output link 2 to the input link 1, M is the mass of the output link 2.

When moving the output link 2 is effected by the inertia force $M\ddot{Z}$, the resistance force F_2 equals $F_2 = H_0$ sign$\dot{Z}$, here H_0 is the coefficient of dry friction, resisting the motion of output link 2.

The force of viscous friction F_3 equals $F_3 = H\dot{Z}$.

When there is the contact and $Y_0 = Y$, the motion of the output link 2 is described by the following equation:

$$M\ddot{Z} + H\dot{Z} + H_0 \operatorname{sign} \dot{Z} + f_0 N \operatorname{sign}(\dot{Z} - \dot{X}) = 0, \qquad (2)$$

here Y_0 is the coordinate of the output link 2 with respect to the input link 1.

The following dimensionless designations are introduced into the formulas:

$$x = \frac{X}{A},\ y = \frac{Y}{A},\ z = \frac{Z}{A},\ 1 = \frac{B}{A},\ h = \frac{H}{\omega M},$$

$$h_0 = \frac{H}{A\omega^2 M},\ f = \frac{F}{A\omega^2 M},\ \tau = \omega\, t. \qquad (3)$$

In the following investigations it is assumed that $\dot{z}>0$.

Then differential Eq. 2 can be written in dimensionless form:

$$\dot{z}'' + hz' + h_0 + f_0(-l\cos\tau + f)\operatorname{sign}(z' - x') = 0 \qquad (4)$$

It is differentiated with respect to the new dimensionless variable τ.

The Eq. 4 of motion of the vibrodrive is solved applying the so called method of junction [4]. In general case for different types of motion two types of slipless contact motion and two types of slipping contact motion are possible.

The motion is of the first type when $\tau = \tau_i,\ z' = x'$.

The second type motion is contact sliding motion, when $\tau \in (\tau_i, \tau_{i+1}),\ z' - x' < 0$.

The coordinate and velocity of the output link 2 are determined from the equation:

$$z'' + hz' = -m + n\cos\tau, \qquad (5)$$

here coefficients $m=h_0-f_0 f$, $n = -f_0\, l$. In this case the input link surpasses the output link 2.

The third type of motion is analogous to the first one i.e. when $\tau = \tau_{i+1},\ z' = x'$.

The fourth type of motion is contact sliding motion when $\tau \in (\tau_{i+1}, \tau_{i+2}),\ z' - x' > 0$.

The coordinate and the velocity of the output link 2 for this type of motion are determined taking into account the following coefficients: $m_1 = h_0 + f_0 \cdot f$, $n_1 = f_0 \cdot l$.

In this case the velocity of the output link 2 is greater than the velocity of the input link 1.

In a general case, for the first and third types of motion i.e., when $\tau = \tau_i$, $\tau = \tau_{i+1}$, the coordinate and the velocity of the output link 2 are determined from the following equations:

$$z_1 = z_0(\tau) + \sin\tau,\ z_i' = \cos\tau. \qquad (6)$$

For the second type of motion, when $\tau \in (\tau_i, \tau_{i+1})$, the coordinate and velocity of the output link 2 are:

$$z_i = C_i - D(\tau - \tau_i) - E\exp[-h(\tau - \tau_i)] + F\sin\tau - G\cos\tau,$$

$$z_i' = -D + hE_i\exp[-h(\tau - \tau_i)] + F\cos\tau + G\sin\tau \qquad (7)$$

and at the end of interval:

$$z_i(\tau_{i+1}) = C_i - D(\tau_{i+1} - \tau_i) - E\exp[-h(\tau_{i+1} - \tau_i)] +$$

$$+F\sin\tau_{i+1} - G\cos\tau_{i+1}, \qquad (8)$$

$$z_i'(\tau_{i+1}) = -B + hE\exp[-h(\tau_{1+i} - \tau_i)] +$$

$$F\cos\tau_{i+1} + G\sin\tau_{i+1},$$

where

$$C_i = \left(\frac{1}{h} - \frac{n_1}{h^2+1}\right)\cos\tau_i + \left[1 + \frac{h^2(n_1 - n) - n}{h(h^2+1)}\right]\sin\tau_i -$$

$$-\left(\frac{m_1}{h^2} - \frac{n_1}{h(h^2+1)}\right)\exp(-h\tau_i) - \frac{m_1}{h}\tau_i + \frac{m + m_1}{h^2},$$

$$E = \left(\frac{1}{h} - \frac{n}{h^2+1}\right)\cos\tau_i - \frac{n}{h(h^2+1)}\sin\tau_i + \frac{m}{h^2},$$

$$D = \frac{m}{h},\ G = \frac{n_1}{h^2+1},\ F = \frac{n_1 h}{h^2+1}. \qquad (9)$$

For the fourth type of motion, when $\tau \in (\tau_{i+1}, \tau_{i+2})$, the coordinate and the velocity of the output link are:

$$z_{i+1} = C_{i+1} - D_1(\tau - \tau_{i+1}) - E_{i+1}\exp[-h(\tau - \tau_{i+1})] +$$

$$+F_1\sin\tau - G_1\cos\tau,$$

$$z_{i+1}' = -D_1 + hE_{i+1}\exp[-h(\tau - \tau_{i+1})] +$$

$$+F_1\cos\tau + G_1\sin\tau, \qquad (10)$$

and at the end of interval:

$$z_{i+1}(\tau_{i+2}) = C_{i+1} - D_1(\tau_{i+2} - \tau_{i+1}) -$$

$$E_{i+1}\exp[-h(\tau_{i+2} - \tau_{i+1})] +$$

$$+F_1\sin\tau_{i+2} - G_1\cos\tau_{i+2},$$

$$z_{i+1}'(\tau_{i+2}) = -D_1 + hE_{i+1}\exp[-h(\tau_{i+2} - \tau_{i+1})] +$$

$$+F_1\cos\tau_{i+2} + G_1\sin\tau_{i+2}, \qquad (11)$$

where

$$C_{i+1} = \left(\frac{1}{h} - \frac{n}{h^2+1}\right)\cos\tau_{i+1} +$$

$$\left[1 + \frac{h^2(n - n_1) - n_1}{h(h^2+1)}\right]\sin\tau_{i+1} -$$

$$-\left[-\frac{m}{h^2} - \frac{h}{h(h\ \ +1)}\right]\exp(-h\tau_{i+1}) - \frac{m}{h}\tau_{i+1} + \frac{m + m_1}{h^2},$$

$$E_{i+1} = \left(\frac{1}{h} - \frac{n_1}{h^2+1}\right)\cos\tau_{i+1} - \frac{n_1}{h(h^2+1)}\sin\tau_{i+1} + \frac{m_1}{h^2},$$

$$D_1 = \frac{m_1}{h},\ G_1 = \frac{n}{h^2+1},\ F_1 = \frac{nh}{h+1}.$$

After a certain number of the output link motion stages the output link 2 performs a steady state motion or approaches this motion regime.

If in time the steady state motion approaches certain limit type motion, the later can be called an asymptotically stable motion. That's why by the so called junction method, which shows the sequence of the steady state motion, it is possible to find out the steady state motion parameters.

It is considered that the steady state motion is determined by the following parameters:

$$\tau_i = \varphi,\ \tau_{i+1} = \varphi + \delta,\ \tau_{i+2} = 2\pi + \varphi,\ z_i = 0,$$

$$z_i = z'_{i+2} = v,\ z_{i+2} = s. \qquad (12)$$

The parameters of the steady state motion φ, δ, v, S are obtained using the solutions of the four type equations of motion.

Taking into account (Eq. 12), Eq. 6÷11 will become:

$$v = \cos\varphi,$$
$$z(\tau) + \sin\varphi = 0,$$
$$z_{i+1} = C_i - D\delta - E_1\, exp(-h\delta) + F\, sin(\varphi+\delta) - G cos(\varphi+\delta)$$
$$z'_{i+1} = -D + hE_i\, exp(-h\delta) + F\, cos(\varphi+\delta) + G\, sin(\varphi+\delta)$$
$$z'_{i+1} = cos(\varphi+\delta),$$
$$z_{i+1} = C_{i+1} - D_1(2\pi-\delta) - E_{i+1}\, exp\left[-h(2\pi-\delta)\right] +$$
$$+ F_1 \sin\varphi - G_1\cos\varphi,$$
$$z'_{i+1} = -C_1 + hE_{i+1}\, exp\left[-h(2\pi-\delta)\right] + F_1\, cos\,\varphi + G_1\, sin\,\varphi,$$
$$z'_{i+2} = v = cos\,\varphi. \qquad (13)$$

From Eq. 13 the equations for the determination of steady state motion parameters are obtained:

$$-D + hE_i \exp(-h\delta) + (F-1)\cos(\varphi+\delta) + G\sin(\varphi+\delta) = 0,$$
$$-D_i + hE_{i+1}\exp\left[-h(2\pi-\delta)\right] + (F-1)\cos\varphi + G_1\sin\varphi = 0,$$
$$v = \cos\varphi,$$
$$S = C_{i+1} - D_1(2\pi-\delta) - E_{i+1}\exp\left[-h(2\pi-\delta)\right] +$$
$$+F_1\sin\varphi - G_1\cos\varphi. \qquad (14)$$

Not all the regimes obtained from Eq. 14 can be applied and not all of them are stable.

The condition of existence of the steady stable motion regime is the possibility to obtain the solution for Eq. 14.

In order to determine which ones of the steady state regimes are stable it is necessary to solve the equations varying the parameters close to the steady state regime. If the solutions of differential equations after parameter variations are going down, such a regime is stable. In the opposite case, not stable.

$$\tau_i = \varphi + \Delta_\varphi, \qquad \tau_{i+1} = \varphi + \delta + \Delta_\delta$$
$$\tau_{i+1} - \tau_i = \delta - \Delta_\delta - \Delta_\varphi, \qquad \tau_{i+2} = 2\pi + \varphi + \rho\Delta_\varphi,$$
$$\tau_{i+2} - \tau_{i+1} = 2\pi + \rho\Delta_\varphi - \delta - \Delta_\delta, \qquad z'_i = v + \Delta_v,$$
$$z'_{i+2} = v + \rho\Delta_v, \qquad z'_i = \Delta_z, \qquad (15)$$

here Δ_φ, Δ_δ, Δ_v, Δ_z are the variations; ρ is the characteristic parameter.

If $|\rho| < 1$, the analyzed regime is stable, if $|\rho| > 1$, – unstable.

The analysis of the equations of motion of the links was performed when the output link is in translation.

Criteria according which the quality of a vibrodrive can be determined are the following: losses of energy because of friction in the contact place of the input and output links, the power of those losses, the useful work of the output link, its average velocity, motions, non-uniformity of motion, etc. These criteria are determined when the vibrodrive motion is in steady state.

Losses in the contact zone:

$$P = f_0(M\ddot{Y} + F)\cdot(\dot{Z} - \dot{X})\mathrm{sign}(\dot{Z} - \dot{X}) \qquad (16)$$

or

$$P = f_0 A^2 M\varpi^2 \cdot p. \qquad (17)$$

here $p = (y^n + f)(z' - x')sign(z' - x')$.

Losses of the energy in the contact zone because of a dry friction are given by:

$$A_{fr} = \int_0^{T_n} Pdt = f_0 A^2 M\omega^2 \alpha_{fr}, \qquad (18)$$

or

$$\alpha_{fr} = \int_0^{\omega T_n} (y^n + f)(z' - x')sign(z' - x')d\tau, \qquad (19)$$

where T_n is the steady state motion period.

The useful work:

$$\mathrm{A_n} = \int_0^{\mathrm{T_n}} \left((H\ddot{Z}) + H_0\ \mathrm{sign}\,\dot{Z}\right)\dot{Z}dt \qquad (20)$$

or

$$\mathrm{A_n} = A^2 M\omega^2\alpha_n, \qquad (21)$$

here

$$\alpha_\mathrm{n} = \int_0^{\omega\mathrm{T_n}} (hz' + h_0\ \mathrm{sign}\,z')z'd\tau.$$

The efficiency is

$$\eta = \alpha_n / \alpha_n + f_0\cdot\alpha_{fr} + \alpha, \qquad (22)$$

where α are the energy losses because of the elastic deformation of the input link. When calculating η, they are neglected, i.e. α=0.

The average velocity

$$\bar{\dot{z}} = \omega' \cdot \bar{z}', \qquad (23)$$

where

$$\bar{z}' = \frac{1}{\omega T_n}\int_0^{\omega T_n} z'd\tau.$$

The non-uniformity coefficient of the motion is given by

$$\delta\dot{z} = \delta z' = (z'_{\max} - z'_{\min})/2z', \qquad (24)$$

where $z'_{\max}, z'_{\min}$ are the maximal and minimal velocities of the output link, respectively.

Dynamic characteristics of the vibrodrive when its motion is in steady state are shown in Fig. 2-6.

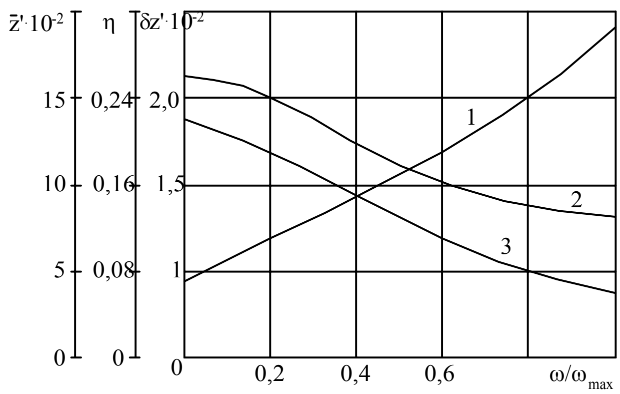

Fig.2. **Dependencies of average velocity $\bar{z}'$, efficiency η, and non-uniformity coefficient of motion δ versus frequency**

($\omega/\omega_{\max}$): 1 – $\bar{z}'$; 2 – η; 3 – $\delta z'$, ψ_0 = 0.01, h=0.05, h_0=1, f=$8\cdot10^{-5}$, f_0=0,05

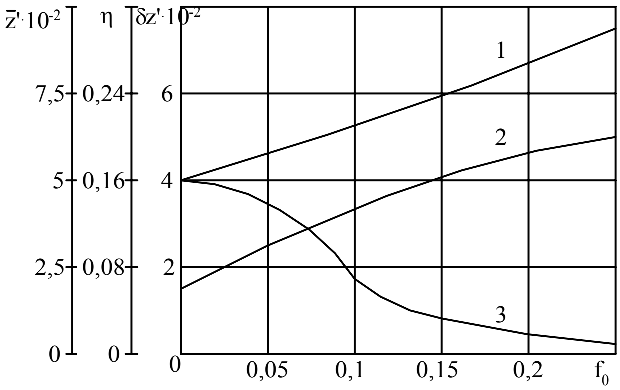

Fig 3 **Dependencies of average velocity** $\bar{z}'$, **efficiency** η, **and non uniformity coefficient of motion** $\delta z't$ **on dry friction coefficient f$_0$: 1 – $\bar{z}'$; 2 – η; 3 – $\delta z'$,** h=0.05, h_0=0,01, f=$8\cdot10^{-5}$, $\omega/\omega_{\max}$=0,5; ψ_0 = 0.01

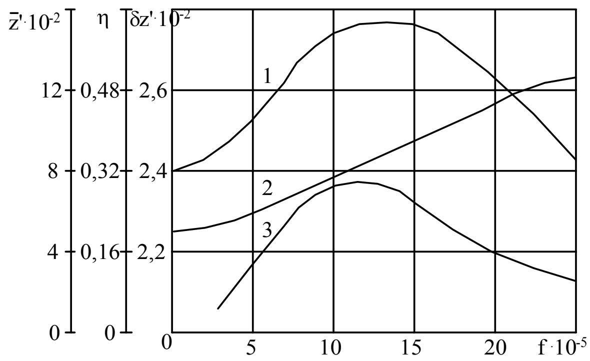

Fig.4. **Dependencies of average velocity** $\bar{z}'$, **efficiency** η, **and non uniformity coefficient of motion** $\delta z'$ **versus pressing force *f*: 1– $\bar{z}'$; 2 – η; 3 – $\delta z'$, ψ_0 =0,01,** h=0.05, h_0=0,1, f_0=0,05, $\omega/\omega_{\max}$ = 0,5

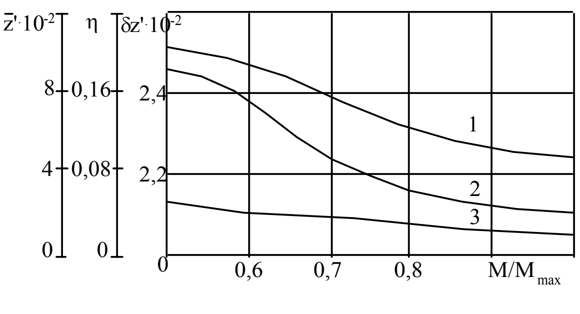

Fig.5. **Dependencies of average velocity** $\bar{z}'$, **efficiency** η, **and non uniformity coefficient of motion** $\delta z'$ **versus the output link mass M/M$_{\max}$: 1 – $\bar{z}'$; 2 – η; 3 – $\delta z'$, ψ_0 =0,01,** h=0.05, h_0=0,1, f_0=0,05, f=$8\cdot10^{-5}$, $\omega/\omega_{\max}$ = 0,5

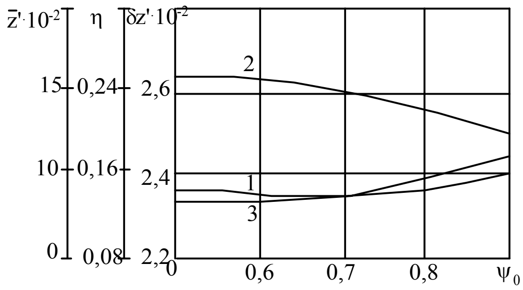

Fig.6. **Dependencies of average velocity** $\bar{z}'$, **efficiency** η, **and non uniformity coefficient of motion** $\delta z'$ **on phase shift** ψ_0**: 1 – $\bar{z}'$; 2 – η; 3 – $\delta z'$,** h=0.05, h_0=0,1, f_0=0,05, f=$8\cdot10^{-5}$, $\omega/\omega_{\max}$ = 0,5.

Conclusions

The wave vibrodrive is presented as a simplified model of a nonlinear oscillating system.

Analytical expressions for the description of a steady state motion of the vibrodrive links are obtained when the links do not rebound from each other. The conditions of the steady state motion existence are indicated. Performing the analysis of differential equations of the links motion the main characteristics of the vibrodrive are determined.

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G. Baurienė, L. Patašinė, K. Pilkauskas

Banginių vibropavarų nusistovėjusio judėsio režimo tyrimas

Reziumė

Pateikti banginės vibropavaros su žiediniu žadintuvu, kontaktuojančiu su išėjimo grandimi tašku, dinamikos analitinių tyrimų rezultatai, esant kinematiniam žadinimui. Netiesinės išėjimo grandies judesio diferiancialinės lygtys išspręstos prijubgimo metodu. Ištirti vibropavaros keturių tipų judesiai: du kontaktiniai neslystamieji ir du kontaktiniai slystamieji judesiai. Gauta transcendentinių lygčių sistema nusistovėjusio judesio parametrams nustatyti.

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